Inequality, Incomplete Contracts, and the Size Distribution of Business Firms

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Abstract

This paper analyzes the effects of intra-firm bargaining on the formation of firms in an economy with imperfect capital markets and contracting constraints. In equilibrium wealth inequality induces a heterogeneous distribution of firm sizes allowing for firms both too small and too large in terms of technical efficiency. The findings connect well to empirical facts such as the missing middle of firm size distributions in developing countries. The model can encompass a non-monotonic relationship between aggregate output and inequality. It turns out that an inflow of capital may indeed decrease output in absolute terms.
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1 Introduction

The size distribution of business firms has proved to be one of the more enduring objects of economic research as it reflects the organization of production in an economy (see e.g. Simon and Bonini, 1958). Naturally, a

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major concern of economic analysis is whether production is organized efficiently, be it within plants, firms, or industries. Within an industry, that is among firms producing the same output good, presumably with access to the same technology, there is little theoretical reason to expect a non-degenerate distribution of sizes in classical competitive equilibrium theory. Empirical evidence, however, points to heterogeneous size distributions of firms. A possible way of reconciliation is that firms differ with respect to unobservable characteristics as, for instance, individual productivity (Lucas, 1978) or attitude towards risk (Kihlstrom and Laffont, 1979), and firm sizes are chosen optimally in response to those characteristics.[1] Yet judging from observations it is far from obvious that all firms are indeed efficiently sized (see e.g. Banerjee and Duflo, 2005).

Moreover, when efficient organization of production given technology equals optimal factor input choice, this primarily delivers a theory of plant size as put forward by Hart (1995). It is not obvious how technical efficiency at plant level connects to technical efficiency at firm level. Integration of technically efficient plants might for instance result in undesirable market concentration. Firm size not only affects production but also output markets or the distribution of profits among stakeholders, an issue this paper emphasizes. That is, to explain firms' choices of size a proper notion of the firm is needed. As a consequence, this paper reverts to the property rights theory in modeling ownership rights on productive assets thereby allowing for well-defined boundaries of the firm.

Departures from a degenerate size distribution have also been attributed to market imperfections or missing markets. Often this comes in conjunction with assumptions on technology that ensure market imperfections on one market affect another. In this paper markets are complete and imperfections are assumed explicitly in that contracts within firms are incomplete, and lead to renegotiations, and there is a spread between lending and borrowing interest rate. Indeed a number of studies provides empirical support for rent sharing in firms (see e.g. Arai, 2003, Hildreth and Oswald, 1997, and the references provided therein), and interest rate spreads exceeding the risk premium appear to be common especially in developing countries (Banerjee, 2003). The production technology is deterministic, has strict

[1] See also the literature on entry and exit of firms in response to technology shocks, e.g. the selection models in Jovanovic (1982) and Hopenhayn (1992).

complementarity between labor and capital, and a unique firm size maximizing average output. This provides a clear-cut efficiency benchmark as opposed to most of the literature where size choice is always constrained efficient given a firm's characteristics, for instance technology or wealth level, and equilibrium prices (as e.g. in Caballero and Hammour, 1998, Cooley et al., 2004).

Agents heterogeneous in initial endowments meet in a one-sided matching market deciding on firm membership, investment in assets, financial position, and instantaneous side payments. Then they engage in productive activities within firms. Due to incompleteness of contracts, agents' payoffs are determined by renegotiations within firms, consistent with intra-firm bargaining in Stole and Zwiebel (1996). Renegotiation payoffs depend positively on asset holdings as ownership of assets improves an agent's outside option in renegotiations. The equilibrium concept is the core with a continuum of agents and endogenous coalition size given continuation payoffs from renegotiation.

The equilibrium firm size distribution is heterogeneous and capable of simultaneously allowing for firms both too small *and* too large compared to the efficient firm size. For this both incomplete contracting and imperfect capital markets are needed. Inequality in initial endowments among agents and capital market imperfections create a wedge in the cost of capital between agents of different wealth levels. Subsequently, poorer agents choose less investment in assets than rich agents do. Renegotiations induce owners of firms to employ more agents than technically efficient as their profit share increases in the number of employees and assets owned. Hence, the firm size desired by a firm owner exceeds the efficient one, yet poorer owners choose smaller sizes. In addition, the model generates a relation of wealth and income inequality reminiscent of the dual economy. Whereas wealth inequality between workers and owners exceeds income inequality, income inequality exceeds wealth inequality between firm members and unmatched agents.

The equilibrium firm size distribution tends to be bimodal when market imperfections and endowment inequality are sufficiently severe and the endowment distribution sufficiently skewed. This is consistent with empirical findings as bimodal size distributions are common in developing countries (e.g. Tybout, 2000), countries plagued by credit market frictions. In contrast, size distributions of firms in developed countries are commonly found to be unimodal (see Cabral and Mata, 2003). Several empirical studies (e.g. Little et al., 1988, Biggs et al., 1995, Hallward-Driemeier et al., 2002) provide some evidence of an inverted U-shaped relationship between efficiency in terms of output per worker and size of firms in countries with bimodal firm size distributions, indicating the presence of both too large and too small firms.

Methodologically this paper is related to Legros and Newman (1996) who consider a one-sided matching market where firms differ in choice of organizational technology. Legros and Newman (2008) find too much integration in a model with two-sided matching of technologically complementary firms into pairs using an incomplete contracting framework. In a partial setting close to Hart and Moore (1990), Bolton and Whinston (1993) find a similar over-integration result. Chakraborty and Citanna (2005) have one-sided matching of pairs of principals and agents and find segregation into occupations due to non-transferabilities induced by limited liability and moral hazard. In contrast to these contributions we employ a one-sided matching framework with non-transferabilities that admits endogenous coalition sizes.

This paper proceeds by introducing a formal model in section 2. Section 3 states useful preliminaries to an equilibrium existence result in section 4. Section 5 studies properties of the equilibrium allocation and Section 6 provides an application of the model, while section 7 concludes.

2 The Model

2.1 Agents

A single-good economy is populated by a continuum of agents living on I, a compact subset of $\mathbb{R}$. The good is used both for consumption and investment. Agents are heterogeneous in their initial endowments of the good given by the mapping $\omega : I \to [\underline{\omega}, \overline{\omega}]$, $0 \leq \underline{\omega} < \overline{\omega}$. $\omega(i)$ is bijective and continuously differentiable. Agent i's utility function $u_i(x_i)$ is linear in consumption of the single good x_i which is given by final payoffs and assumed to be $u_i = x_i$.

2.2 Production

Production occurs in firms and uses labor and capital as inputs. Capital and labor are strict complements in the sense that each unit of labor needs an investment c to be productive. Capital investment is discrete and fully depreciates at the end of the period. This is best understood as investment in productive assets such as machines. Denote the production function by $f(k,n)$, where k is the capital invested and n denotes the number of firm members. Strict complementarity then is captured by the assumption $f(k,n) = \min\{f(nc,n); f(k,k/c)\}$ for $n > 1$. Utility maximization by agents will preclude waste of resources and therefore we will drop capital and simply write $f(n)$ to denote production using nc units of capital and n units of labor for $n > 1$. Assumptions on $f(n)$ are as follows.

Assumption 1 *(Production technology)*

(i) $f(0,1) = f(c,1) = f(1) = w_0$.

(ii) $f(n) - f(n-1) > f(n+1) - f(n) \geq 0 \ \forall n \geq 2$.

(iii) $\lim_{n\to\infty}(f(n) - f(n-1)) \leq w_0$.

Assumption (i) specifies an outside option w_0 of agents who remain unmatched. w_0 is best thought of as subsistence income from manual labor. Technology may embody team production, that is locally increasing returns when moving from the outside option to team production, that is $f(1) - 0 < f(2) - f(1)$. By Assumption (ii) the production technology has strictly diminishing returns to scale for firms of size $n > 1$ and Assumption (iii) guarantees that coalition sizes are finite. Note that Assumptions (ii) and (iii) ensure that there is a firm size $K \geq 1$ that maximizes average surplus.

Example: Suppose that $f(n) = \hat{f}(n) + nw_0$ *with* $\hat{f}(n) = (n-1)^{1-\frac{1}{K}}$, $K \in \mathbb{N}$, $K > 1$, *and* $w_0 > 0$. *Then* $f(n)$ *satisfies Assumption 1 and average output,* $f(n)/n$, *is maximized at* $n = K$.

2.3 Firms and Ownership

Output is produced jointly by agents within finitely sized coalitions, which can be understood as partnerships, or firms. Within a coalition of agents each member matters, but the impact of a team on the whole economy is negligible. Let $\mathcal{F}(I)$ denote the set of finite subsets of I. A coalition will be denoted by $N \in \mathcal{F}(I)$ consisting of $n = |N|$ members.[2]

The key idea of this model is that agents may own capital in a firm. For expositional purposes suppose that only two regimes of ownership are possible: an agent either owns all capital in a firm or no capital at all. Ownership is acquired by investing the single good in capital either using initial endowments or borrowing on the capital market. Call an agent who has invested in and owns the firm's productive assets the *manager* M, to indicate that M can exclude other agents. An agent not owning any capital is called a *worker* W. Hence, a *firm* is given by a set of agents N consisting of a manager M and a set of workers W. This setup precludes outside ownership and external finance is limited to personalized debt.

Of course, the framework can be generalized considerably by explicitly modeling productive assets and distributions of ownership rights on those assets. This requires heavy notation, however, and the interested reader is referred to the working paper version (Gall, 2005).

2.4 Capital Market

There exists a capital market enabling agents to borrow or lend. This market is imperfect in that there is an interest rate spread between lending interest rate $1 + r$ and borrowing interest rate $1 + i \equiv (1 + r)\gamma$.[3] $\gamma > 1$ is an exogenous parameter capturing the severity of capital market frictions in the economy. Note that $\gamma = 1$ implies perfect markets which precludes the formation of inefficient firms (see section 5). The spread may be generated by moral hazard on the borrower's side or recovery cost in case of default. The lending interest rate $1+r$ is exogenous since agents live in a small open economy.[4]

To render the analysis non-trivial assume that $K > 1$ and that production in firms is indeed efficient at least for some sizes $2 \leq n \leq K + 1$.

Assumption 2 $f(2) > 2(w_0+(1+r)c)$ *and* $f(K+1)-f(K) > w_0+(1+r)c$.

[2] In the course of this paper a coalition N is referred to by the number of its members, n, and the term n firm is used as a synonym.

[3] This is a simple and straightforward way to model imperfect credit markets that has frequently been used in the literature, see e.g. Galor and Zeira (1993) for a similar setup.

[4] Endogenizing the lending rate r does not alter our results, see the working paper version.

Because of deceasing differences also $f(n) > n(w_0+(1+r)c)$ for $n = 2, ..., K+1$. Part (iii) of Assumption 1 implies that

$$\frac{f(\tilde{n})}{\tilde{n}} > w_0 + (1+r)c > \frac{f(\tilde{n}+1)}{\tilde{n}+1} \tag{1}$$

defines a unique firm size $\tilde{n}$ such that $f(n) > n(w_0 + (1+r)c) \Leftrightarrow n \leq \tilde{n}$. Hence, $\tilde{n}$ is a finite upper bound on firm sizes consistent with profit maximization.

2.5 First Best Benchmark

Let us provide an efficiency benchmark for comparison with later results. In a first best world, where capital markets are perfect and contracts complete, coalition size maximizes the marginal increase in net output of an additional worker. This is, given the structure of the production function, equivalent to maximizing average output per coalition member. Hence, in a first best world all firms in the economy are of size K.

2.6 Sequence of Events

The timing of events in the model economy is given as follows.

1) Matching market opens, agents simultaneously decide on coalitions, investment plans, and their capital market position.

2) Production takes place and firm members may renegotiate the distribution of profits within firms at any time.

3) Payoffs take place.

2.7 Contractual Environment and Renegotiations

Labor contracts are incomplete in the sense that they are non-binding *at the matching market stage*. For instance, they could be renegotiated at any time before production has finalized. This is the case when there are problems of contract enforcement due to weak legal institutions, or when opportunities arise for agents to hide away output. This describes an economy where written labor contracts or worker protection laws are not common.

Individual profit shares in firm N, $\pi_i(N)$ result from renegotiations between firm members after the matching stage. The specific bargaining protocol is not crucial to our results. Required properties of renegotiation payoffs are

(i) Renegotiation does not waste resources, i.e. $\sum_{i \in N} \pi_i(N) = f(n)$.

(ii) Surplus is split in an $n = 2$ firm, i.e. $\pi_i(N) = \pi_j(N)$ for all $N = \{i, j\}$.

(iii) Workers' renegotiation payoffs strictly decrease in firm size n for at least all sizes $n \leq K$.

These properties are consistent with several well-known extensive-form bargaining games.[5] The key property is that workers' renegotiation payoffs diminish while average output increases.[6] We focus on the case where individual profit shares depend only on the size of a firm and the role an individual has in the firm (manager or worker). Thus $\pi_i(N)$ reduces to payoffs $\pi_W(n)$ for a worker and $\pi_M(n)$ for the manager in a size n firm. This precludes settings where an individual's outside option in renegotiations is to default.

Intrafirm Bargaining

Consider a specific bargaining model for illustration, intra-firm bargaining proposed by Stole and Zwiebel (1996). They require payoffs in a firm to be stable, such that no player gains from initiating renegotiations between a worker and the manager. Worker-manager renegotiations equally split the *additional* surplus these two agents obtain from cooperation. Define a payoff profile in firm N as a collection $\{(\pi_W(N_j), \pi_M(N_j))_{M \in N_j \subseteq N : |N_j| = j}, j = 2, .., |N|\}$ with $j\pi_W(N_j) + \pi_M(N_j) = f(j)$, that is payoffs for the manager and any subset of workers in the firm such that the payoffs are feasible, i.e. sum up to joint output for each subset. A payoff profile thus specifies payoffs for all deviating coalitions involving the manager.

[5] Apart from the bargaining game considered in Stole and Zwiebel (1996) this is true e.g. for the version in Westermark (2003) and for the renegotiations in Hart and Moore (1990).

[6] This property is needed for our results. Suppose instead that the manager makes take it or leave it offers to workers and captures the workers' entire surplus net of the outside option given by the market wage. Since the labor market is one-sided, the single market wage makes a marginal agent indifferent between the roles of worker and manager of a firm of at least size K due to team production. Hence, only $n \geq K$ firms will emerge.

Definition 1 (Stable Payoff Profile) *A payoff profile in firm N is stable if for all $j = 2, .., |N|$ no agent $i \in N_j$ with $|N_j| = j$, $M \in N_j \subseteq N$ can improve upon $\pi_i(N_j)$ in a worker-manager renegotiation.*

A stable payoff profile ensures that for all subsets N_j of firm members there is no agent who gains by initiating renegotiations. Stole and Zwiebel (1996) present an extensive form non-cooperative bargaining game that supports the stable payoff profile as the outcome associated to a Nash equilibrium.

Determine now joint surplus in a firm N. At the renegotiation stage the matching market is closed. Renegotiation takes place among members within each firm. The manager has the right to block other agents' access to firm capital, i.e. to fire workers. Agents' outside options may depend on their capital market position and on their ownership rights. Denote agent i's net position on the capital market by R_i if i lends and by D_i if i borrows. Applying for a job in another firm is not feasible since the market is closed at the time of renegotiation and replacing a worker in an existing firm does not increase the renegotiation payoff of that firm's manager. The joint surplus is thus given as

$$\Pi(N) = f(n) + \sum_{i \in N} [(1+r)R_i - (1+i)D_i - \max\{w_0 + (1+r)R_i - (1+i)D_i; 0\}],$$

since borrowers may default when leaving the firm. The following assumption is made to ensure that the subsistence income w_0 is always sufficient to repay an agent's debt.[7]

Assumption 3 (No Default) *Subsistence income is high enough, $w_0 \geq \frac{f(n)}{n+1}$ for all $n \leq \tilde{n}$.*

With full depreciation this implies that investment cost c is small compared to output and outside option. Note that $\pi_W(N) - (1+i)D_i \geq w_0 + (1+r)\omega(i)$ must hold to induce a borrower i to work in an N firm, hence $\pi_W(N) > w_0$ for borrowers. Workers who lend obtain at least w_0 from renegotiations since this is their outside option. Since $\pi_M(N) \leq f(n) - (n-1)w_0$ and $\pi_M(N) - (1+i)D_i \geq w_0 + (1+r)\omega(i)$, the assumption implies no default:

[7] This ensures that profit shares do not depend on individual characteristics. Otherwise the set of continuation payoffs (i.e. occupations) may become a continuum. Since we use an induction argument to prove uniqueness of the equilibrium allocation, allowing for default in equilibrium introduces technical complications while not adding interesting results.

$(1+i)D_i \leq \pi_M(N) - w_0 \leq w_0$. An analogous expression holds for workers who borrow. Then

$$\Pi(N) = f(n) - nw_0.$$

Additional surplus to be split in worker-manager renegotiation between worker i and manager M at N is $(\pi_M(N) + \pi_i(N)) - (\pi_M(N \backslash \{i\}) + w_0)$. Hence, $\pi_M(2) = \pi_W(2) = f(2)/2$ for all $|N| = 2$ with $M \in N$. Payoffs for higher firm sizes can then be derived by induction as in Stole and Zwiebel (1996).[8]

Proposition 1 *Under intrafirm bargaining in a firm N with $n \leq \tilde{n}$ the stable payoff profiles are given by*

$$\begin{aligned}
\pi_M(n) &= \frac{1}{n}\sum_{i=2}^{n}(f(i) - (i-1)w_0) + \frac{w_0}{n}, \\
\pi_W(n) &= \frac{f(n)}{n} - \frac{w_0 + \sum_{i=2}^{n-1} f(i)}{n(n-1)} + \frac{w_0}{2}.
\end{aligned}$$

It holds that (i) $\pi_M(n) + (n-1)\pi_W(n) = f(n)$, (ii) $\pi_W(2) = \pi_M(2) = f(2)/2$, and (iii) $\pi_W(n) \geq w_0$ and $\pi_W(n)$ strictly decreases in n for $n \leq \tilde{n}$.

Proof: The expression for π_W and π_M follow from a straightforward application of the proof of Theorem 1 in Stole and Zwiebel (1996) accounting for the manager's outside option w_0. Properties (i) and (ii) are obvious, for (iii) note that $\pi_W(n) \geq w_0$ since $f(n)/n > w_0$ for all $n \leq \tilde{n}$ and

$$\pi_W(n+1) - \pi_W(n) = \frac{\sum_{i=1}^{n-1}(n+1-i)(n-i)[f(n+2-i) - 2f(n+1-i) + f(n-i)]}{(n+1)n(n-1)},$$

where we use $f(1) = w_0$. Decreasing differences of $f(n)$ then implies (iii). $\square$

Note that $\pi_M(n) - \pi_M(n-1) = \pi_W(n) - w_0$ so that a manager's payoff strictly increases in firm size for $n \leq \tilde{n}$. The manager's profit share $\pi_M(N)/f(n)$ increases at least for $n \leq K$ since workers' payoffs decrease and average output increases while output is shared in $n = 2$ firms.

[8]The present version differs only in that the manager (the firm in their terminology) has outside option w_0 if all workers quit.

Example: Using the production function $f(n) = (n-1)^{1-\frac{1}{K}} + nw_0$ *we obtain*

$$\begin{aligned}
\pi_M(n) &= w_0 + \frac{\sum_{i=2}^{n}(i-1)^{1-\frac{1}{K}}}{n} \quad \textit{and} \\
\pi_W(n) &= w_0 + \frac{(n-1)^{1-\frac{1}{K}}}{n} - \frac{\sum_{i=2}^{n-1}(i-1)^{1-\frac{1}{K}}}{n(n-1)}.
\end{aligned}$$

In general, a production function of the form $f(n) = \hat{f}(n) + nw_0$*, such that* $\hat{f}(1) = 0$*,* $\hat{f}(n)$ *increases and has decreasing differences in* n*, satisfies* $\pi_M(n) \geq \pi_W(n) > w_0$ *for* $n > 1$*. Assumptions 2 and 3 imply* $w_0 > (\tilde{n} - 1)^{1-\frac{1}{K}} > \tilde{n}(1+r)c$*.*

2.8 The Matching Market

On the matching market individuals simultaneously decide on the firm, if any, they wish to join, whether to be worker or manager in that firm, any monetary side payments to be paid or received, and on the associated investment in capital and their capital market position. Denote an individual's occupational choice by (θ, N), that is i's role $\theta \in \{W; M\}$ in firm N. The lending interest rate $1+r$ is given exogenously. At the matching stage agents anticipate their continuation payoffs from choosing a role in a firm N given by the individual payoffs from renegotiations $\pi_M(n)$ for a manager or $\pi_W(n)$ for a worker.

Let $v(\omega(i), \theta, N, t(i), r)$ denote agent i's valuation for a choice of firm and role. The valuation depends on individual wealth because of the capital market friction, on monetary side payments $t(i) \in \mathbb{R}$ i receives when choosing a role, and on the interest rate r. In case agent i plans to be manager, which requires an investment of nc to hold ownership,

$$v(\omega(i), M, N, t(i), r) = \pi_M(n) + (\omega(i) + t(i) - nc)\begin{cases}(1+r) & \text{if } \omega(i) + t(i) \geq nc, \\ (1+i) & \text{if } \omega(i) + t(i) < nc.\end{cases}$$

In case i plans to be worker $v(.)$ is given by

$$v(\omega(i), W, N, t(i), r) = \pi_W(n) + (\omega(i) + t(i))\begin{cases}(1+r) & \text{if } \omega(i) + t(i) \geq 0, \\ (1+i) & \text{if } \omega(i) + t(i) < 0.\end{cases}$$

Denote agent i's outside option by $v(\omega(i), W, 1, 0, r) = w_0 + (1+r)\omega_i$ corresponding to occupational choice $(W, 1)$. It is given by the subsistence income and lending on the capital market. So equipped we now define a non-transferable utility matching equilibrium with side payments.

Definition 2 *A matching equilibrium with side payments denoted by* (P^*, θ^*, t^*) *is a measure consistent partition* P^* *of the agent space, a collection of individual role choices* θ^**, and side payments* t^* *such that*

(i) $\sum_{i \in P} t^*(i) = 0$ *for all* $P \in P^*$ *(budget balance of side payments),*

(ii) $\nexists P_i' \in \mathcal{F}(I)$ *such that* $v(\omega(i), \theta', P_i', t'(i), r) > v(\omega(i), \theta^*(i), P_i^*, t^*(i), r)\ \forall\ i \in P_i',\ i \in P_i^*$ *and* $\theta' \in \{W; M\}$ *such that* $\sum_{j \in P_i'} t'(j) = 0$ *(stability).*

Budget balance ensures feasibility of side payments. Measure consistency intuitively requires that the measure of first members of any coalition must equate the measure of the second members which must equate the measure of third members etc.[9] This equilibrium concept postulates that there does not exist any blocking coalition which is feasible with respect to the distribution of ownership rights and aggregate endowments and makes every member strictly better off possibly involving side payments. It coincides with the f-core with limited side payments (see Kaneko and Wooders, 1996).

3 Preliminaries

At this point it is convenient to characterize properties that any equilibrium allocation must satisfy. Stability in particular puts some structure on side payments. Note that equilibrium side payments are only defined for firms that are actually formed in equilibrium. The following lemma states some useful properties of equilibrium side payments.

Lemma 1 (Equilibrium Side Payments) *Stability implies that*

(i) equilibrium side payments depend only on firm size and individual ownership, that is $t(i) = t(\theta, n)$*,*

(ii) equilibrium side payments in $n = 2$ *firms are* $t(M, 2) = -t(W, 2) = c$*,*

(iii) equilibrium side payments to workers are strictly increasing in n*, that is* $t(W, n) > t(W, n')$ *for all* $n > n'$*.*

[9]See Kaneko and Wooders (1986) for the formal definition and extensive discussion.

Proof: In Appendix.

That is, side payments in firms with positive equilibrium measure permit interpretation as market prices for accessing continuation payoffs in that firm. Since $\pi_M(2) = \pi_W(2)$, upfront investment is split equally in $n = 2$ firms. Furthermore, $t(W, 2) = -c$ is a lower bound for side payments to workers since workers' side payments increase in firm size n. That is, bigger firms pay higher side payments and lower continuation payments, which may e.g. resemble expected continuation income when promotion is stochastic.[10] Intuitively, labor market payments need to increase in firm size to compensate for the deterioration of an individual worker's bargaining position as the firm size grows.

Turn now to individual investment choice given side payments t and the interest rate r. An agent i prefers to be manager of an n firm to being manager of an $n' < n$ firm if

$$\pi_M(n) + (1+r)(\omega(i)+t(M,n)-nc) \geq \pi_M(n') + (1+r)(\omega(i)+t(M,n')-n'c),$$

provided $\omega(i)$ suffices to fund upfront investment $nc - t(M, n)$. That is,

$$\pi_M(n) - \pi_M(n') \geq (1+r)(t(M,n') - t(M,n) + (n - n')c),$$

which implies that all individuals with sufficient endowments choose the same investment. The same reasoning applies to individuals who have to borrow for both investment alternatives. Suppose now n requires higher upfront investment and $n'c - t(M, n') < \omega(i) < nc - t(M, n)$ for some agent i. i prefers more investment, that is n, if

$$\pi_M(n) + (1+i)(\omega(i)+t(M,n)-nc) \geq \pi_M(n') + (1+r)(\omega(i)+t(M,n')-n'c).$$

This is a condition on endowments $\omega(i)$:

$$\omega(i) \geq n'c + \frac{\pi_M(n') - \pi_M(n)}{(1+r)(\gamma-1)} + \frac{\gamma[(n-n')c - t(M,n)] + t(M,n')}{\gamma - 1} \equiv \omega^*(M,n,M,n').$$

[10] Present value of life time labor income depends on individual cost of capital. Discounted at the borrowing rate $1 + i$ labor income strictly increases in firm size whereas the reverse is true when discounting at the lending rate $1 + r$. Connecting this to empirical findings may be difficult in absence of further worker heterogeneity, e.g. in productivity.

Indeed it can be verified that $n'c - t(M,n') < \omega^*(M,n,M,n') < t(M,n) - nc$ implies that all $i \in I$ with $\omega(i) < \omega^*(M,n,M,n')$ prefer n' to n, and all $i \in I$ with $\omega(i) > \omega^*(M,n,M,n')$ prefer n to n'. If $n'c - t(M,n') \geq \omega^*(M,n,M,n')$, all individuals $i \in I$ prefer n to n' and if $nc - t(M,n) \leq \omega^*(M,n,M,n')$ all individuals $i \in I$ prefer n' to n. The following lemma generalizes this observation.

Lemma 2 (Investment Behavior) *Let side payments be fixed. For any pair (θ, n) and (θ', n'), $\theta, \theta' \in \{M, W\}$, requiring investments $nc - t(M,n)$ if $\theta = M$ $(-t(W,n)$ if $\theta = W)$ and $n'c - t(M,n')$ if $\theta' = M$ $(-t(W,n')$ if $\theta' = W)$, there is at most one endowment level $\omega^* \in [\underline{\omega}, \overline{\omega}]$, such that*

- *an agent endowed with ω^* is indifferent,*
- *an agent with $\omega(i) > \omega^*$ prefers the tuple requiring higher investment,*
- *an agent with $\omega(i) < \omega^*$ prefers the other one.*

ω^* is non-decreasing in r.

Proof: In Appendix.

That is, richer agents prefer occupations requiring higher upfront investments since the credit market friction gives them an absolute advantage in form of lower cost of capital. Note also that owning a bigger firm requires higher upfront investment than owning a smaller firm or working:

$$n > n' \Rightarrow nc - t(M,n) \geq n'c - t(M,n'), \tag{2}$$

with strict inequality if $n > 2$ using balanced side payments and statements (ii) and (iii) of Lemma 1. That is, monotonicity of investment choice in upfront investment net of side payments translates into monotonicity of investment choice in firm size. Hence, wealthier individuals prefer owning bigger firms at equilibrium side payments. Noting that managers in the model are in fact entrepreneurs, Lemma 2 implies that entrepreneurship is positively correlated with wealth. Changes in the interest rate affects investment behavior in the expected direction: an interest rate increase shifts investment preferences towards occupational choices requiring lower upfront investments.

Recall that firm sizes in equilibrium have a finite bound $\tilde{n}$ given by (1). This permits to partition the agent set into finitely many subsets of agents with the same optimal choice of investment. Figure 1 gives an example of such a partition depending on the interest rate r. The support of the endowment distribution is on the vertical axis and the interest rate on the horizontal axis. The curves show tuples (r, ω) making an agent with wealth ω indifferent between the labeled occupations. Figure 1 shows agents' preferred occupational choices at given side payments (here all set to zero so that the partition represents the technology and the renegotiation outcome). This describes a class society of agents as found in the works by Evans and Jovanovic (1989) and Banerjee and Newman (1993). Varying side payments shifts curves up or down, changing the measures of agents choosing each occupation. Equilibrium side payments have to align measures of owners and workers in proportion to the respective firm size, and thus the equilibrium partition depends on the wealth distribution.

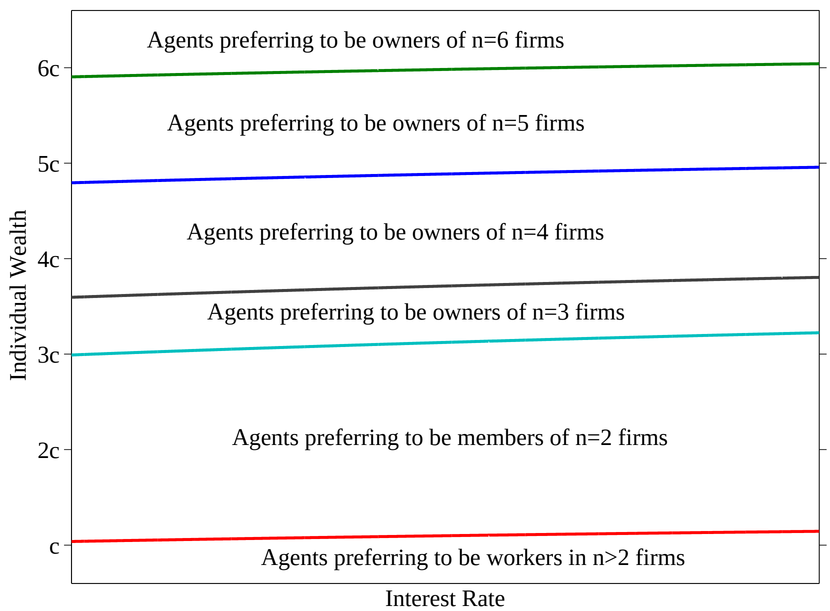

Figure 1: Ownership preferences depending on endowments and interest rate

4 Existence

Start by noting that the matching pattern can be described as a necessary condition for any equilibrium. Matching must be coarsely negative assortative in equilibrium, that is the bigger the firm the poorer the workers and the richer the managers.

Proposition 2 (Matching Pattern) *In equilibrium the agent space can be partitioned into agent classes $C(\theta,n)$ such that for all $i \in C(\theta,n)$ i chooses the tuple (θ,n) and $\omega(i) \in [\underline{\omega}(\theta,n),\overline{\omega}(\theta,n)]$. For any firm size $n > 2$ with positive measure in equilibrium it must hold that*

(i) $\overline{\omega}(W,n) \leq \underline{\omega}(M,n)$,

(ii) for any two firm sizes $n' < n$ with positive measure in equilibrium $\overline{\omega}(W,n) < \underline{\omega}(W,n')$ *and* $\underline{\omega}(M,n) > \overline{\omega}(M,n')$.

Proof: Construct the partition using cutoff values $\omega^*(\theta,n,\theta',n')$. With (2), $[\underline{\omega},\overline{\omega}]$ can be partitioned into classes $C(\theta,n)$ such that all $i \in C(\theta,n)$ choose (θ,n). By Lemma 2 cutoffs $\omega^*(\theta,n,\theta',n')$ are unique for bilateral comparison, so that classes $c(\theta,n)$ are representable as intervals of $[\underline{\omega},\overline{\omega}]$. By Lemma 1 managers' upfront investment is strictly higher than workers' for $n > 2$ firms and equal for $n = 2$ firms yielding statement (i). (ii) follows as side payments for workers decrease in firm size from Lemma 1, by Lemma 2 and (2) firm size choice increases (decreases) in own endowments for managers (workers). □

Verify now that a matching equilibrium exists and induces a unique firm size distribution. The full proof is somewhat involved and can be found in the appendix but a sketch follows. Existence of a matching equilibrium is implied by a well known result of Kaneko and Wooders (1996). Exploiting monotonicity of agents' preferences we show that coalition size and allocation of ownership rights are determined uniquely almost everywhere. To do so we exploit the segmented labor market wages analogy of side payments in firms with positive measure in equilibrium. This means excess labor demand can be defined for every firm size. Given side payments t, let μ_{n_M} be the Lebesgue measure of agents weakly preferring (M,n) to all $(\theta',n') \neq (M,n)$ with firm size n' having positive measure:[11]

$$\mu_{n_M} = \mu(\{i \in I : v(\omega(i),M,n,t(M,n),r) \geq v(\omega(i),\theta',n',t(\theta',n'),r)\}). \quad (3)$$

[11]Note that side payments in firms with measure zero in equilibrium are not defined. However, due to monotonicity found in Lemma 2, if there exist side payments $t(.,n)$ for a firm size n such that $\mu_{n_M} = \mu_{n_W} = 0$, then this firm size must have measure zero. Therefore a side payment $t(.,n)$ with the above property can be used as shadow price for firm types with measure zero which is convenient for numerical simulation.

Define μ_{n_W} likewise as the measure of agents weakly preferring (W, n) to all $(\theta', n') \neq (W, n)$ with firm size n' having positive measure:

$$\mu_{n_W} = \mu(\{j \in J : v(\omega(j), W, n, t(W, n), r) \geq v(\omega(j), \theta', n', t(\theta', n'), r)\}). \quad (4)$$

Denote by μ_n the measure of firms of size n in the economy. Stability and measure consistency imply that for all firm sizes $n > 1$

$$\mu_n \leq \mu_{n_M} \text{ and } (n-1)\mu_n \leq \mu_{n_W}. \quad (5)$$

Moreover, there may be agents preferring to remain unmatched. By Proposition 2 there exists a cutoff endowment ω_U defined by

$$\omega_U = \frac{w_0 - \pi_W(\overline{n})}{(1+r)(\gamma-1)} - \frac{\gamma}{\gamma-1} t(W, \overline{n}), \quad (6)$$

where $\overline{n} = \max n$ s.t. $\mu_n > 0$ denotes the biggest firm size with positive measure. That is, interpreting unmatched agents employed in the subsistence technology as unemployed, unemployment depends on scarcity of workers in the labor market for $\overline{n}$ firms, and thus on the endowment distribution. Finally, measures of firms must satisfy the accounting identity

$$\sum_{n=1}^{\overline{n}} n\mu_n = \mu(i : \omega(i) \geq \max\{\omega_U, \underline{\omega}\}).$$

By Proposition 2 matching is negative assortative in classes such that prospective workers become poorer and managers richer as the firm size increases. Intuitively, this allows sequential clearing (descending in firm size) of each labor market (3), (4) and (5) by adjusting side payments *in the subsequent market* allowing to construct an aggregate excess labor demand. Uniqueness follows from monotonicity of investment behavior in upfront investment cost.

Proposition 3 (Existence) *A matching equilibrium with limited side payments exists and induces a unique distribution of firm sizes.*

Proof: In Appendix.

To illustrate the mechanism at work consider a simple example. Suppose an economy is characterized by high endowment inequality and severe credit market imperfections. In particular, let a large measure of agents with zero endowments face a comparatively small measure of individuals endowed with some positive amount of wealth. Although not covered by the assumption of continuous differentiability of $\omega(i)$, an equilibrium may exist nevertheless. Suppose the measure of poor agents exceed the one of the rich more than $K+1$-fold. Then only large $n > K$ firms emerge, owned by the rich, and all poor individuals are workers.

5 Firm Size and Endowment Distribution

This section presents some general properties of the equilibrium size distribution, notably that firms heterogeneous in size may emerge.

5.1 Too Small and too Large Firms

Key to equilibrium analysis is that all agents obtain at least the payoff from being member of a symmetric firm, i.e. $v(\omega(i), M, 2, c, r) = v(\omega(i), W, 2, -c, r)$, or the outside option $v(\omega(i), W, 1, 0, r) = w_0 + (1+r)\omega(i)$, whichever is higher. This is because all agents can choose size 2 firms not needing agents from other endowment classes. Therefore an agent chooses to work if and only if this is better than the outside option and the cost of investing at least c is sufficiently high. Likewise, to be owners agents must prefer this to the outside option and have sufficiently low cost of capital for investing at least c.

It is useful to define the difference in workers' renegotiation payoffs between firm sizes l and m as

$$\Delta\pi_W(l, m) = \frac{f(l)}{l} - \frac{f(m)}{m} + \frac{w_0 + \sum_{i=2}^{l-1} f(i)}{l(l-1)} - \frac{w_0 + \sum_{i=2}^{m-1} f(i)}{m(m-1)}.$$

Recall that renegotiation payoffs increase in firms size for owners and decrease for workers by Proposition 1 and thus $\Delta\pi_W(l, m) > 0$ if $l < m$ and $\Delta\pi_W(l, m) < 0$ if $l > m$. This induces inefficient incentives for occupational choice that may not be fully mitigated by side payments due to heterogeneous cost of capital. As a consequence firm formation may not be efficient, that is there may emerge $n \neq K$ firms. Since the paper focuses on the formation of inefficiently sized firms, the following proposition provides a necessary condition for efficient firm formation.[12]

Proposition 4 *The equilibrium firm size distribution is degenerate at size K only if*

$$\frac{f(K)}{K} > \alpha(n)\frac{f(n)}{n} + (1-\alpha(n))\frac{f(2)}{2} + \frac{\gamma-1}{\gamma}\frac{K-1}{K}\Delta\pi_W(K,n)\,\forall\, n>K,\ \textit{and}$$
$$\frac{f(K)}{K} > \beta(n')\frac{f(n')}{n'} + (1-\beta(n'))\frac{f(2)}{2} + (\gamma-1)\frac{n'-1}{K}\Delta\pi_W(n',K)\,\forall\, n'<K,$$

where $\alpha(n) = (nK-n)/(nK-K) < 1$ *and* $\beta(n') = n'/K < 1$.

Proof: In Appendix.

Intuitively, firm size is chosen based on a trade-off between output efficiency and efficiency of the utility transfer scarce agents have to receive by means of side payments. For this trade-off both within firm renegotiations and capital market imperfections are needed. Renegotiations pin down surplus shares at $\pi_W(n)$ and $\pi_M(n)$. To obtain surplus shares other than that instantaneous side payments are needed – which are costly due to imperfect capital markets. Suppose for the moment utility is perfectly transferable by setting $\gamma = 1$. This eliminates the trade-off and results in the formation of only efficient firms.

Proposition 4 is a condition on the production function that requires K firms to be sufficiently more productive than other firm sizes (comparing the LHS of the necessary condition to the first two terms on the RHS) as to compensate the difference in cost of capital for financing side payments (the third term on the RHS). Inspecting the inequalities it turns out that there always exists a degree of capital market imperfection that implies positive measure of too small firms, yet this is not the case for too large firms. The following corollary gives a sufficient condition for one-sided inefficiency.

Corollary 1 *If either condition of Proposition 4 holds while the other one fails, there is one-sided inefficiency, that is a positive measure of either too small or too large firms emerges in equilibrium.*

In order to have both too small and too large firms a more elaborate condition is needed.

[12] A straightforward sufficient condition for efficient firm formation is that γ is sufficiently close to 1. This makes utility perfectly transferable so that coalitions maximize joint surplus.

Proposition 5 *If the support of the endowment distribution is sufficiently large and the conditions in Proposition 4 fail for $n' = K-1$ and $n = K+1$ and*

$$\frac{f(K+1)}{K+1} \geq \frac{K(K-1)\frac{f(K-1)}{K-1} - 2\frac{f(2)}{2}}{(K+1)(K-2)} - \frac{\gamma-1}{\gamma}\frac{K}{K+1}\Delta\pi_W(K-1, K+1) \text{ and}$$
$$\frac{f(K+1)}{K+1} \leq \frac{(K-1)\frac{f(K-1)}{K-1} + 2\frac{f(2)}{2}}{K+1} + (\gamma-1)\frac{K-2}{K+1}\Delta\pi_W(K-1, K+1),$$

and for all firm sizes $h' < K-1$ and $\overline{n} > h > K+1$

$$\frac{f(K-1)}{K-1} \geq \frac{h'\frac{f(h')}{h'} + (K-1-h')\frac{f(2)}{2}}{K-1} + (\gamma-1)\frac{h'-1}{K-1}\Delta\pi_W(h', K-1) \text{ and}$$
$$\frac{f(K+1)}{K+1} \geq \frac{Kh\frac{f(h)}{h} + (h-K-1)\frac{f(2)}{2}}{(K+1)(h-1)} + \frac{\gamma-1}{\gamma}\frac{K}{K+1}\Delta\pi_W(K+1, h),$$

then both size $K-1$ and $K+1$ firms have positive measure in equilibrium.

Proof: In Appendix.

The proposition gives sufficient conditions for both too small and too large firms emerging simultaneously in equilibrium. The first part requires $K+1$ and $K-1$ firms not to be strictly preferred to each other by all agents, the second condition ensures $K+1$ and $K-1$ firms are preferred to all other firms by a positive measure of agents. Here we consider only $K+1$ and $K-1$ firms to save on notation. However, the result holds generally: for any inefficient firm size $n \neq K$ to have positive measure in equilibrium, market imperfections have to be sufficiently severe and average output in n firms has to be sufficiently high compared to (but needs not exceed that of) other firm sizes.

Example: Consider again the technology $f(n) = (n-1)^{1-\frac{1}{K}} + nw_0$ and determine whether it allows for both size $K{+}1$ and $K{-}1$ firms in equilibrium. First, Proposition 4 must fail for those firm sizes:

$$\frac{\gamma-1}{\gamma}\Delta\pi_W(K, K+1) \geq (K-1)^{-\frac{1}{K}} - K^{-\frac{1}{K}} - \frac{1}{2K(K-1)}, \text{ and}$$
$$(\gamma-1)\Delta\pi_W(K-1, K) \geq \frac{K-1}{K-2}(K-1)^{-\frac{1}{K}} - (K-2)^{-\frac{1}{K}} - \frac{1}{2(K-2)}.$$

It can be shown by induction on K that $\Delta\pi_W(K, K{+}1) > (K{-}1)^{-\frac{1}{K}} - K^{-\frac{1}{K}} - (2K(K-1))^{-1}$ for all $K > 2$. Hence, there always exists γ sufficiently large *so that the inequalities above hold. Choosing for instance $K = 4$ and $\gamma = 2.5$ satisfies these conditions, and also those in Proposition 5 (see Appendix).*

5.2 Large Firms and Endowment Redistribution

A neat feature of Propositions 4 and 5 is that they are distribution-free, except for a condition on the upper bound of the support. They state conditions on the primitives that imply for certain firm sizes there always exist side payments so that some agents strictly prefer to work in and others to manage such firms. For a given individual an occupation's access cost and payoff depend on labor market side payments. Relative scarcity of potential managers compared to potential workers affects side payments. Since individual cost of capital for an occupation depends on wealth, firm size and endowment distributions are linked. For instance, if endowments are distributed very equally with mean c, potential managers abound and wages are high. This makes increasing firm size beyond the efficient level more expensive. We now characterize this relationship considering particular changes of the endowment distribution, namely rotations.[13]

Definition 3 *Let F, G be endowment distributions with common mean μ_ω. G is a counterclockwise rotation of F around the rotation point $\hat{\omega}$ if*

$$G(\omega) \geq F(\omega)\, \forall\, \omega < \hat{\omega},\ G(\hat{\omega}) = F(\hat{\omega}),\ \text{and } G(\omega) \leq F(\omega)\, \forall\, \omega > \hat{\omega}.$$

This describes redistributions of endowments from the rich to the poor as the density of agents poorer than $\hat{\omega}$ decreases when moving from F to G. Note that F is a mean preserving spread of G, and in this sense G is more equal. Denote by $\hat{n}$ the firm size an agent with $\hat{\omega}$ is member of in equilibrium under F.

Proposition 6 *Let F be an endowment distribution with $\underline{\omega}^F < c < \overline{\omega}^F$. There exist endowments $\hat{\omega} \in [c, \overline{\omega}^F)$ and $\omega' < \omega'' \in [\underline{\omega}^F, c)$ such that for a counterclockwise rotation G of F around $\hat{\omega}$ with $F(\omega'') - F(\omega') \geq G(\omega'') - G(\omega')$ the aggregate measure of $n > \hat{n}$ firms, the largest firm size $\overline{n}$ with positive measure, and the measure of unmatched agents are weakly smaller in an equilibrium under G than under F.*

[13] See for instance Johnson and Myatt (2006) for a discussion of the concept.

Proof: In Appendix.

Redistributing endowments from potential owners to potential workers has a twofold effect: a direct effect decreasing the supply of owners of (and possibly of workers in) large firms, and an indirect effect increasing side payments to owners of large firms due to the change in labor demand and supply. The direct effect is certain to dominate if the labor supply for large firms decreases when moving to G. It is given by the agents with $\omega(i) \in [\omega_U, \omega^*(W,\hat{n},W,\hat{n}^+)]$ under F as defined in (6) and (10), $\hat{n}^+ > \hat{n}$ denoting the next higher firm size. If given F all agents match into firms (that is $\omega_U \leq \underline{\omega}$), any counterclockwise rotation around some $\hat{\omega} > c$ has the property.

That is, suitable redistributions of initial endowments from owners to workers exist that prevent the formation of inefficiently large firms and increase the number of agents employed in the industrial sector (see also the example in the next section). Unfortunately, the general relationship between output efficiency and wealth inequality is ambiguous. Note e.g. that in the limit, for a degenerate endowment distribution, all agents must obtain the same payoff. Therefore the firm size distribution is degenerate, at the efficient size K if capital markets work well enough (i.e. γ sufficiently close to 1), or aggregate wealth exceeds c, and otherwise at size 2 enabling equal sharing of surplus without side payments. Moreover, the impact on aggregate output of a change in the wealth distribution depends on the subsequent change of the entire firm size distribution unlike e.g. in Grüner (2003).

5.3 The Income Distribution

Of particular interest is whether inequality is amplified or dampened by economic activity, that is whether endowment inequality exceeds income inequality. Note that an agent's income is given by $v(\omega(i), \theta^*, n^*, t(\theta^*, n^*), r)$. Suppose for the moment that $\underline{\omega} > 0$ to properly define the endowment gap for agents i and j, $\omega(i) > \omega(j)$, as $\omega(i)/\omega(j)$. Define the income gap between the same agents i and j likewise as $v(\omega(i), .)/v(\omega(j), .)$. Recall that ω_U is defined by (6) as the cutoff endowment that separates unmatched agents, with $\omega(i) < \omega_U$ and matched agents, with $\omega(i) > \omega_U$.

Proposition 7 *An agent's income is weakly increasing in endowments.*

$$\frac{v(\omega(i),.)}{v(\omega(j),.)} > \frac{\omega(i)}{\omega(j)} \quad \textit{for } \omega(i) > \omega_U > \omega(j) > 0 \textit{ and}$$
$$\frac{v(\omega(i),.)}{v(\omega(j),.)} < \frac{\omega(i)}{\omega(j)} \quad \textit{for } \omega(i) > \omega(j) > \omega_U.$$

Proof: In Appendix.

That is, income inequality in terms of ratios is smaller than endowment inequality among matched agents. Hence, there is scope for convergence of income in a dynamic version of the model. This does not extend to agents remaining unmatched as the gap between poorest and richest agent widens, creating a dual economy flavor.

6 Application

6.1 Bimodal Size Distributions

Firm size distributions in developing countries are typically characterized by a missing middle (see e.g. Tybout, 2000, Sleuwaegen and Goedhuys, 2002). That is, the size distribution of firms is bimodal. Other studies (e.g. van Biesebroeck, 2003) find that the share of the work force employed in intermediately sized firms is significantly less than both the shares of those employed in small or large firms. In contrast, developed economies typically have skewed, unimodal distributions of workforce per firm size (see e.g. Cabral and Mata, 2003).

This provides an opportunity to apply the model. Suppose that the endowment density is at most single peaked, increases before attaining the peak at $\hat{\hat{\omega}} < c$ and decreases thereafter. To obtain a bimodal firm size distribution some small and some large firm sizes must be more attractive to both owners and workers than some intermediate size. This requires sufficient measure of agents with endowments inducing them to be members of small and large firms and thus depends on the endowment distribution.

Example: We now limit profitable firm sizes to 2, 3, *and* 4 *by assuming* $f(n) = \hat{f}(n) + nw_0$ *with* $\hat{f}(n) = (n-1)^{1-\frac{1}{K}}$ *for* $n \leq 4$ *and* $\hat{f}(n) = 0$ *otherwise. Let* $K = 3$ *and* $\omega(i)$ *be uniformly distributed on* $[\underline{\omega}, \overline{\omega}]$*. Abbreviate payoffs* *by $t(n) = -t(W,n) = t(M,n)/(n-1)$. Cutoff endowments for workers are*

$$\begin{aligned}
\omega^U &= \frac{\gamma t(4)}{\gamma - 1} - \frac{3\hat{f}(4) - \hat{f}(3) - \hat{f}(2)}{12(1+r)(\gamma-1)} \\
\omega^*(W,3,W,4) &= \frac{\gamma t(3) - t(4)}{\gamma - 1} + \frac{3\hat{f}(4) - 5\hat{f}(3) + \hat{f}(2)}{12(1+r)(\gamma-1)} \\
\omega^*(W,2,W,3) &= \frac{\gamma c - t(3)}{\gamma - 1} + \frac{\hat{f}(3) - 2\hat{f}(2)}{3(1+r)(\gamma-1)}.
\end{aligned}$$

For owners we have

$$\begin{aligned}
\omega^*(M,3,M,4) &= 3c + \frac{\gamma c + 2t(3) - 3\gamma t(4)}{\gamma - 1} - \frac{3\hat{f}(4) - \hat{f}(3) - \hat{f}(2)}{12(1+r)(\gamma-1)} \\
\omega^*(M,2,M,3) &= c + \frac{2\gamma(c - t(3))}{\gamma - 1} - \frac{\hat{f}(3) - \hat{f}(2)}{6(1+r)(\gamma-1)}.
\end{aligned}$$

Assume that $4c-3t(4) > \overline{\omega} > \omega^*(M,3,M,4) > 3c-2t(3) > \omega^*(M,2,M,3) > c\omega^*(W,2,W,3) > t(3) > \omega^*(W,3,W,4) > t(4) > \underline{\omega} > \omega_U$, that is all agents are matched, all firm sizes have positive measure, and the measure of indifferent agents is zero.[14] *Then equalizing labor demand and supply implies*

$$\begin{aligned}
\frac{\omega^*(W,3,W,4) - \underline{\omega}}{\overline{\omega} - \underline{\omega}} &= 3\frac{\overline{\omega} - \omega^*(M,3,M,4)}{\overline{\omega} - \underline{\omega}}, \\
\frac{\omega^*(W,2,W,3) - \omega^*(W,3,W,4)}{\overline{\omega} - \underline{\omega}} &= 2\frac{\omega^*(M,3,M,4) - \omega^*(M,2,M,3)}{\overline{\omega} - \underline{\omega}}.
\end{aligned}$$

Solving these two equations yields equilibrium side payments $t(3)$ and $t(4)$:

$$\begin{aligned}
t(3) &= \frac{(63\gamma^2 - 7\gamma - 5)c - (6\gamma^2 - 5\gamma - 1)(3\overline{\omega} + \underline{\omega}) - \frac{(9\gamma+3)\hat{f}(4) - (15\gamma+1)\hat{f}(3) + (39\gamma+3)\hat{f}(2)}{12(1+r)}}{39\gamma^2 + 13\gamma - 1}, \\
t(4) &= \frac{(59\gamma^2 + 13\gamma - 21)c - 5(\gamma^2 - 1)(3\overline{\omega} + \underline{\omega}) - \frac{(27\gamma+12)\hat{f}(4) + (7\gamma-8)\hat{f}(3) - (13\gamma-22)\hat{f}(2)}{12(1+r)}}{39\gamma^2 + 13\gamma - 1}.
\end{aligned}$$

Choosing parameters $\gamma = 3$, $r = 0.1$, $c = 0.18$, $\underline{\omega} = 0.05$, $\overline{\omega} = 0.30$ we find that the above assumption holds indeed. At equilibrium transfers $t(3) = 0.1362$ and $t(4) = 0.1180$ the size distribution is bimodal: $\mu_2 = 0.2549$, $\mu_3 = 0.0408$, and $\mu_4 = 0.0920$.

Let now the wealth distribution rotate counterclockwise around $\hat{\omega} = 0.175 <$

[14] This is not typically satisfied, but greatly facilitates the exposition as the equilibrium allocation can be obtained by solving a system of equations rather than weak inequalities.

c and set $\underline{\omega}=0.05+\epsilon$, $\overline{\omega}=0.30-\epsilon$. Since $\omega_U<0.05$ the assumption of Proposition 6 holds despite $\hat{\omega}<c$. In a new equilibrium with $\epsilon=0.01$ the measures of size $n>2$ firms have decreased: $\mu_2=0.2664$, $\mu_3=0.0381$, and $\mu_4=0.0882$ at side payments $t(3)=0.1382$ and $t(4)=0.1200$.

Intuitively, agents rich enough to invest in size $n>2$ firms are scarce in the example since $\overline{\omega} \ll 3c$, whereas poor agents abound, $\underline{\omega} \ll c$. This drives up side payments in $n>2$ firms. Moreover, the difference in average output between efficient size 3 and size 4 firms is small due to the production technology. Combined this means some potential size 3 owners are induced to own size 4 firms. On the other hand, the choice of distribution places large measure on endowments that induce agents to invest $c \approx (\overline{\omega} - \underline{\omega})/2$ and match into size 2 firms. The next proposition states a general necessary distributional property.

Proposition 8 *A bimodal equilibrium firm size distribution implies that (i) $\overline{\omega} - \underline{\omega}$ is sufficiently large, (ii) there exist $c<\omega<\omega'<\omega''$ such that*

$$\frac{\mu(\omega \leq \omega(i) \leq \omega')}{\omega' - \omega} < \kappa_1 \frac{\mu(\omega' \leq \omega(i) \leq \omega'')}{\omega'' - \omega'},$$

and/or (iii) there exist $\omega<\omega'<\omega''<c$ such that

$$\kappa_2 \frac{\mu(\omega \leq \omega(i) \leq \omega')}{\omega' - \omega} > \frac{\mu(\omega' \leq \omega(i) \leq \omega'')}{\omega'' - \omega'}.$$

$0 < \kappa_1, \kappa_2 < \infty$ are constants depending on $f(n)$, c, γ, and r.

Proof: In Appendix.

This means that given a technology lower bounds on the slope of the endowment density in at least one of its tails can be constructed, in that sense requiring a skewed wealth distribution. Loosely speaking we need either a Paretian right tail or the peak to sit in the left tail. Figure 2 shows a more sophisticated numerical example of an adequately skewed wealth distribution generating a bimodal firm size distribution (see the appendix for simulation details). Reducing endowment inequality while keeping all else equal leads to a single peaked firm size distribution in Figure 3. The darker bars in the figures represent the size distribution of firms and the lighter bars depict the workforce distribution. The efficient firm size is $K=4$, so that output is higher for the size distribution in Figure 3.

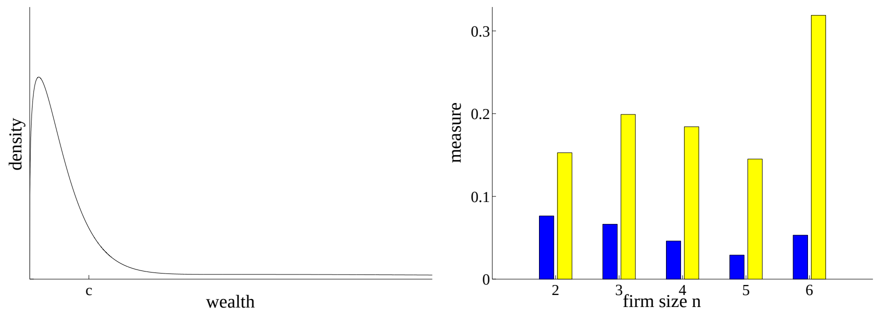

Figure 2: Wealth distribution leading to a bimodal size distribution

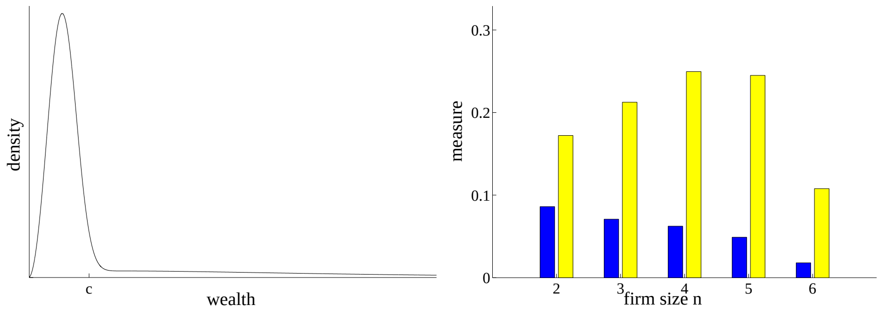

Figure 3: Wealth distribution leading to a unimodal firm size distribution

As the mode of the endowment distribution shifts to the right while preserving the mean, probability mass shifts away from the tails which may result in a loss of the right tail's Paretian properties. This decreases the measure of agents who choose occupations in the largest firm and eliminates the second peak in the firm size and workforce distributions.

6.2 Increasing Aggregate Endowments

Efficiency and the shape of the firm size distribution also depend on aggregate wealth. If aggregate endowments increase sufficiently, agents become less heterogeneous in their cost of capital. This points to a beneficial role for foreign aid and direct investments. Yet this intuition is incomplete as adverse distributional effects may dominate production augmenting effects of a capital influx.

Consider the following thought experiment. Suppose aggregate endowments increase while holding constant investment cost c, such that both $\underline{\omega}$ and the skewness of the endowment distribution increase. The efficient firm size is again $K = 4$.[15] Endowment densities are depicted to the left in Figure 4 and the corresponding equilibrium workforce distributions to the right. The darker bars represent the allocation under lower aggregate endowments. The dashed line represents the initial endowment distribution and the endowment distribution after the increase in wealth is depicted by the solid line.

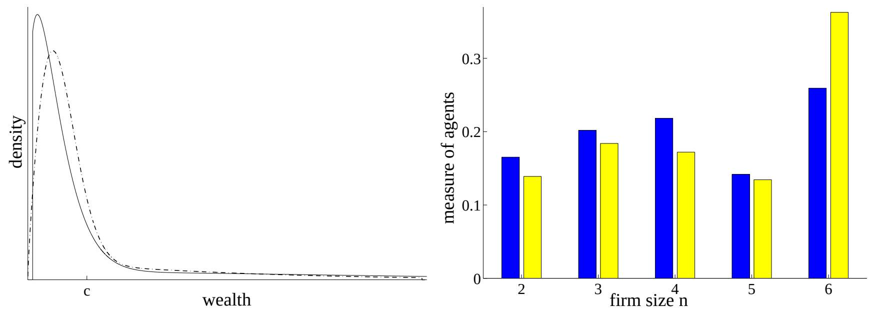

Figure 4: An increase in wealth leading to a decrease in income

In the initial endowment distribution, workers abound and owners are scarce. If aggregate wealth increases sufficiently, this reverses. If aggregate endowments increase, but not enough (as in Figure 4), workers remain abundant whereas the measure of owners increases and thus more agents match into inefficiently large firms, and the measure of efficient K firms decreases. Hence, the distributional effect of a capital increase may indeed dominate the direct effect on aggregate output.[16] This provides a caveat to foreign aid and foreign direct investment, since in particular the latter amounts to an influx of new agents with low capital cost, thus boosting the right tail of the endowment distribution. Therefore economies with high wealth inequality and scarce endowments need not benefit from an increase of the capital stock unless the inflow is sufficiently large or adequately distributed.

7 Conclusion

The paper presented a model of firm formation when capital markets are imperfect and labor contracts non-binding. The equilibrium allocation induces

[15] See the appendix for details of the numerical simulation.

[16] In case of an endogenous interest rate this is partly mitigated by a decrease of the interest rate, though not sufficiently so as to prevent the case depicted (see working paper version).

a unique firm size distribution and permits interpretation as a segmented labor market. Heterogeneity in capital cost induces a heterogeneous firm size distribution with richer agents becoming managers and poorer agents becoming workers. Matching is negative assortative in wealth, that is the wealthier a manager is the larger is his firm while the reverse is true for workers.

It is noteworthy that discrete investment in capital can be interpreted as any kind of productive asset. For instance, capital investments might represent plants connecting lower cost of capital for managers to empire building. Similarly, managers can be viewed as downstream units that may over-engage in multi-sourcing. Size of a firm can also be interpreted as its brand variety. Then the variety of goods produced in an economy depends on wealth inequality and more severe capital market imperfections, which suffice for the emergence of too small but not too large firms, may translate into less variety.

How do the findings fit into the big picture of development economics? In developing economies the labor force distribution among firm sizes is frequently found to be bimodal, and in that way to exhibit a missing middle. This is not known for industrialized economies. The model is able to explain this empirical fact as bimodal firm size distributions may emerge for skewed endowment distributions with a Paretian right tail. As endowment inequality decreases and the mode shifts to the right the firm size distribution becomes unimodal. An increase in endowments does not necessarily lead to an increase in output, especially if the poor abound and extra endowments are not distributed exactly as to induce additional demand for ownership of efficient firms. Moreover, the model generates a wedge in income of matched and unmatched agents thus potentially allowing for dynamics of a dual economy. This suggests that the model provides an adequate instrument for policy analysis of the industrial sector in developing countries.

An obvious extension of this work may analyze the effects of the development of contractual and labor institutions, for instance by introducing cooperatives or collective bargaining. Here arises a neat connection to the work of Williamson (2000) proposing a theory of developing institutions where secure property rights emerge before contract enforcement.

A Mathematical Appendix

Proof of Lemma 1

(i) Suppose $t(i,\theta,N_i) \neq t(j,\theta,N_j)$ for some θ, $|N_i| = |N_j| = n > 1$ with positive measure. Suppose $t(i,\theta,N_i) < t(j,\theta,N_j)$ and $N_i \neq N_j$ without loss of generality. Then there exists a blocking coalition $N' = \{i\} \cup N_j \setminus \{j\}$. That is, there is $\epsilon > 0$ with $n\epsilon = t(j,\theta,N_j) - t(i,\theta,N_i)$ such that

$$\begin{aligned} v(\omega(i),\theta,t(i,\theta,N_i)+\epsilon,N',r) &> v(\omega(i),\theta,t(i,\theta,N_i),N_i,r) \text{ and} \\ v(\omega(k),\theta(k),t(k,\theta(k),N_j)+\epsilon,N',r) &> v(\omega(k),\theta(k),t(k,\theta(k),N_j),N_j,r) \end{aligned}$$

for all $k \in N_j \setminus \{j\}$, a contradiction to stability. Hence, $t(i,\theta,N_i) = t(j,\theta,N_j)$ with $|N_i| = |N_j|$ for all $i,j \in I$ with $\theta(i) = \theta(j) = \theta$. Statement (i) follows.

(ii) Suppose $\mu_2 > 0$ and $t(M,2) \neq -t(W,2)$. Let $t(M,2) > -t(W,2)$ without loss of generality. Then it can be shown as above that an agent i, $\theta(i) = W$ in an $n = 2$ firm, has a blocking coalition with some agent $j \neq i$, $\theta(j) = W$ in an $n = 2$ firm. Hence, $t(M,2) \neq -t(W,2)$ contradicts stability and statement (ii) follows.

(iii) Suppose the contrary and let firm sizes n, $n' < n$ have positive measure and $t(W,n) < t(W,n')$. An n worker with endowment $\omega(i)$ prefers working in an n firm to an n' firm if

$$\begin{aligned} \pi_W(n) + (1+r)(\omega(i) + t(W,n)) &> \pi_W(n') + (1+r)(\omega(i) + t(W,n')) \text{ or} \\ \pi_W(n) + (1+r)\gamma(\omega(i) + t(W,n)) &> \pi_W(n') + (1+r)(\omega(i) + t(W,n')) \text{ or} \\ \pi_W(n) + (1+r)\gamma(\omega(i) + t(W,n)) &> \pi_W(n') + (1+r)\gamma(\omega(i) + t(W,n')), \end{aligned}$$

corresponding to the cases $\omega(i) > -t(W,n) > -t(W,n')$, $-t(W,n) > \omega(i) > -t(W,n')$, and $-t(W,n) > -t(W,n') > \omega(i)$. That is

$$\begin{aligned} \pi_W(n) - \pi_W(n') &> (1+r)(t(W,n') - t(W,n)) \text{ or} \\ \pi_W(n) - \pi_W(n') &> (1+r)[(\gamma t(W,') - t(W,n)) - (\gamma - 1)\omega(i)] \text{ or} \\ \pi_W(n) - \pi_W(n') &> (1+r)\gamma(t(W,n') - t(W,n)). \end{aligned}$$

This yields a contradiction in all three cases noting that by Proposition 1 the LHS of all inequalities is negative. That is, a blocking coalition can be constructed as in the cases above and monotonicity of $t(W,n)$ is verified. $\square$

Proof of Lemma 2

Denote the upfront investment for occupational role (θ, N) by $I(\theta, n) = nc - t(M, n)$ if $\theta = M$ and $I(\theta, n) = -t(W, n)$ if $\theta = W$. Consider roles (θ, n) and (θ', n'). Let $I(\theta, n) > I(\theta', n')$ without loss of generality. Individual i with $\omega(i)$ prefers (θ, n) to (θ', n') if

$$v(\omega(i), \theta, N, t(\theta, n), r) \geq v(\omega(i), \theta', N', t(\theta', n'), r). \tag{7}$$

In case $\omega(i) > I(\theta, n)$ this is

$$\begin{aligned} \pi_\theta(n) + (1+r)(\omega(i) - I(\theta, n)) &\geq \pi_{\theta'}(n') + (1+r)(\omega(i) - I(\theta', n')) \\ \Leftrightarrow \pi_\theta(n) - \pi_{\theta'}(n') &\geq (1+r)(I(\theta, n) - I(\theta', n')). \end{aligned} \tag{8}$$

In case $\omega(i) < I(\theta', n')$ this is

$$\begin{aligned} \pi_\theta(n) + (1+i)(\omega(i) - I(\theta, n)) &\geq \pi_{\theta'}(n') + (1+i)(\omega(i) - I(\theta', n')) \\ \Leftrightarrow \pi_\theta(n) - \pi_{\theta'}(n') &\geq (1+i)(I(\theta, n) - I(\theta', n')). \end{aligned} \tag{9}$$

In case $I(\theta', n') \leq \omega(i) \leq I(\theta, n)$ (7) becomes

$$\begin{aligned} &\pi_\theta(n) + (1{+}i)(\omega(i) - I(\theta, n)) \geq \pi_{\theta'}(n') + (1{+}r)(\omega(i) - I(\theta', n')) \\ \Leftrightarrow \quad &\omega(i) \geq \frac{\pi_{\theta'}(n') - \pi_\theta(n)}{(1{+}r)(\gamma{-}1)} + \frac{\gamma I(\theta, n) - I(\theta', n')}{\gamma{-}1} \equiv \omega^C(\theta,n,\theta',n'). \end{aligned}$$

The cutoff endowment $\omega^C(\theta,n,\theta',n')$ is unique for each pair of (θ, n) and (θ', n'), although not necessarily $\omega^C(.) \in [\underline{\omega}, \overline{\omega}]$. Some algebra yields the implications

$$\omega^C(\theta, n, \theta', n') > I(\theta, n) \Rightarrow \neg(8) \Rightarrow \neg(9),$$

implying that (7) does not hold for all $i \in I$. Analogously,

$$\omega^C(\theta, n, \theta', n') < I(\theta', n') \Rightarrow (9) \Rightarrow (8),$$

implying that (7) does hold for all $i \in I$. Finally,

$$I(\theta', n') \leq \omega^C(\theta, n, \theta', n') \leq I(\theta, n) \Rightarrow (8) \wedge \neg(9),$$

implying (7) holds for all $\omega(i) \geq \omega^C(.)$ and does not hold for all $\omega(i) < \omega^C(.)$. Thus we can construct $\omega^*(\theta, n, \theta', n')$ as

$$\omega^*(\theta,n,\theta',n') = \begin{cases} \underline{\omega} & \text{if } \omega^C(\theta, n, \theta', n') < I(\theta', n') \\ \overline{\omega} & \text{if } \omega^C(\theta, n, \theta', n') > I(\theta, n) \\ \omega^C(\theta, n, \theta', n') & \text{otherwise.} \end{cases} \quad (10)$$

Hence, there is a unique cutoff endowment $\omega^*(.)$ for comparison of any two occupational roles such that the role requiring higher investment cost $I(.)$ is preferred by all $i \in I$ with $\omega(i) > \omega^*(.)$ and the other one by all $i \in I$ with $\omega(i) < \omega^*(.)$. However, agents with $\omega(i) = \omega^*(.)$ need not be indifferent if $\omega^*(.) = \underline{\omega}$ or $\omega^*(.) = \overline{\omega}$. Moreover, $\omega^*(\theta,n,\theta',n')$ increases in $t(\theta, n)$, decreases in $t(\theta', n')$ and increases in r, all strictly if both (θ, n) and (θ', n') are chosen by a positive measure of agents. □

Proof of Proposition 3

We first establish existence of a matching equilibrium given an interest rate r. Then we show that the equilibrium firm size distribution is unique.

Existence of the Matching Equilibrium

The proof of existence follows Legros and Newman (1996). A modified version of θ, θ^M, is needed to construct a super-additive characteristic function of a game (I, θ, v) along the lines of Shubik and Wooders (1983). Define a modified $\theta^M(N)$, $N \in \mathcal{F}(I)$ as follows:

$$\theta^M : \mathcal{F}(I) \rightarrow \{M, W, \emptyset\}^{\mathbb{R}} \times [0, 1],\ \theta^M(N) = ((\theta(i))_{i \in N}, q(N)),$$

where $q(N) \neq q(O) \Leftrightarrow N \neq O$, with $N, O \in \mathcal{F}(I)$. $q(N)$ specifies an index of the organizational unit N, that is firm N. Define the feasible roles in firm N as $\Theta(N) = \{(\theta(i))_{i \in N} : |\{i \in N : \theta(i) = M\}| = 1, |\{i \in N : \theta(i) = W\}| = |N| - 1 \text{ for } i \neq N, \theta(i) = W \text{ for } i = N\}$. Define the individual valuation excluding side payments v^M as $v^M(\omega(i), \theta^M(N), r) = v(\omega(i), \theta(i), N, 0, r)$. Now let $V(O)$ with $O = \bigcup_k O_k$, where $O_k \in \mathcal{F}(I)$ are disjoint finite sets of agents, denote the characteristic function of the economy (I, θ^M, v):

$$V(O) = \{(v^M(\omega(i), \theta_M(O_k(i)), r))_{i \in O} : \theta^M(O_k) \in (\Theta(O_k), q(N))\ \forall O_k \subseteq O\},$$

where $O_k(i)$ denotes the subset $O_k \subseteq O$ with $i \in O_k$. $V(O)$ describes the set of agents' attainable payoff vectors in coalitions $O_k \subseteq O$ achievable by choosing ownership rights allocations.[17] Let $o = |O|$. Note that any union of disjoint coalitions can use the same allocation as the disjoint coalitions. Then construct the comprehensive extension of $V(O)$ by defining

$$\hat{V}(O) = \{x \in \mathbb{R}^o : x \leq V(O)\}.$$

$\hat{V}(O)$ has the following properties:

$$\hat{V} \text{ is a non-empty, closed subset of } \mathbb{R}^o \, \forall \, O \in \mathcal{F}(I), \tag{11}$$

$$\hat{V}(O) \times \hat{V}(O') \subseteq \hat{V}(O \cup O') \, \forall \, O, O' \in \mathcal{F}(I), \tag{12}$$

$$\inf \sup \hat{V}(\{i\}) > -\infty, \tag{13}$$

$$\forall \, O \in \mathcal{F}(I), x \in \hat{V}(O) \text{ and } y \in \mathbb{R}^o \text{ with } y \leq x \Rightarrow y \in \hat{V}(O), \tag{14}$$

$$\forall \, O \in \mathcal{F}(I), \hat{V}(O) - \bigcup_{i \in O} (\operatorname{int} \hat{V}(\{i\})) \times \mathbb{R}^{o-1} \text{ is non-empty and bounded.} \tag{15}$$

Properties 11, 12 and 14 follow directly by definition. Property 13 follows from the existence of an outside option, $V(\{i\}) \geq 0$. This and the definition of $\hat{V}$ also imply property 15. Therefore $\hat{V}$ is a characteristic function in the sense of Kaneko and Wooders (1986).

Represent all agents $i_1, i_2, ..., i_p \in O$ by their wealth $\omega(i_k)$. Then it is straightforward that also

$$\begin{aligned} \hat{V}(\omega(i_{\rho(1)}), \omega(i_{\rho(2)}), ..., \omega(i_{\rho(p)})) &= \{(x_{\rho(1)}, x_{\rho(2)}, ..., x_{\rho(p)}) : (x_1, x_2, ..., x_p) \\ &\quad \in \hat{V}(\omega(i_1), \omega(i_2), ..., \omega(i_p))\} \end{aligned}$$

for all permutations ρ of O. Thus the conditions Comprehensiveness (property 13), Nontriviality (implied by property 15), and Anonymity in Kaneko and Wooders (1996) hold. By expression (1) coalition sizes are bounded above by $\tilde{n}$. It remains to show continuity of $\{x \in \mathbb{R}^n : V(\{i\}) \leq x \leq V(O)\}$ on $[\underline{\omega}, \overline{\omega}]^n$ for $n = 1, ..., \tilde{n}$ which holds by definition.

Since agents may use side payments to transfer utility at a strictly positive rate we may apply the Theorem of Kaneko and Wooders (1996). Thus existence of the f-core of the characteristic function game associated with

[17] Note that the notation using O is equivalent to a notation using the corresponding vector of attributes $(\omega_1, \omega_2, ..., \omega_n)$ as in Kaneko and Wooders (1996).

$\hat{V}$ follows. It remains to show that an allocation in the f-core of $\hat{V}$ is also an equilibrium as in Definition 2. An allocation in the f-core of $\hat{V}$ for some $O \in \mathcal{F}$ gives rise to payoffs $\hat{x} \in \hat{V}(O)$ that cannot be improved upon in the sense of stability. For $x \in V(O)$ it must hold that $x \geq \hat{x}$ by construction of $\hat{V}$. Then for the equilibrium allocation neither can x be improved upon. Finally, by definition of $V(.)$ for all $x \in V(O)$ and $\hat{x} \in \hat{V}(O)$ such that $\hat{x} \leq x$ there exist disjoint subsets of O, O_k, a mapping θ^M, and side payments t such that $x_i = v(\omega(i), \theta^M(O_k(i))(i), O_k(i), t(i), r)$. Define for all $j \in [0,1]$ $O_j = \{i \in I : \theta^M(i) = (\theta(i), j)\}$. Then the collection $(O_j, j \in [0,1] : O_j \neq \emptyset)$ defines the equilibrium coalitions in the sense of our equilibrium definition.

Uniqueness of the Matching Equilibrium

Now we show that for any matching equilibrium the equilibrium measures of firms $(\mu_n)_{n:\mu_n>0}$ are unique almost everywhere. We first proof uniqueness of side payments consistent with stability and measure consistency. Then we show that μ_n is uniquely determined by side payments almost everywhere.

Step 1: Uniqueness of side payments. Define measures $\mu_{n_M}^{strict}$ and $\mu_{n_W}^{strict}$ as measures of agents strictly preferring to be owner of n firms, or worker in n firms, respectively, to all other roles in firms with positive measure in equilibrium. An equilibrium vector of side payments t induces stability and measure consistency which imply jointly

$$\mu_{n_M}^{strict} \leq \mu_n \leq \mu_{n_M} \text{ and } \mu_{n_W}^{strict} \leq n\mu_n \leq \mu_{n_W}. \tag{16}$$

By Lemma 1, definitions (3), (4) and for the strict versions accordingly, measures μ_{n_M}, μ_{n_W} and $\mu_{n_M}^{strict}$, $\mu_{n_W}^{strict}$ are fully characterized by side payments $t(n) = t(M,n) = -(n-1)t(W,n)$. By inspection of the endowment cutoff values $\omega_M(n,.)$ and $\omega_W(n,.)$ taken from the proof of Lemma 1, measures μ_{n_W} and $\mu_{n_W}^{strict}$ strictly decrease in $t(n)$ for $\mu_{n_W}, \mu_{n_W}^{strict} \in (0,1)$ and measures μ_{n_M}, $\mu_{n_M}^{strict}$ strictly increase in $t(n)$ for $\mu_{n_M}, \mu_{n_M}^{strict} \in (0,1)$. Moreover, if $\mu_{n_M}|_{t(n)} > \mu_{n_M}^{strict}|_{t(n)}$ for $t'(n) \neq t(n)$, all other side payments equal, $\mu_{n_M}|_{t'(n)} = \mu_{n_M}^{strict}|_{t'(n)}$ and likewise for μ_{n_W} and $\mu_{n_W}^{strict}$. Hence, some μ_n consistent with t is not consistent with t' such that $t(j) = t'(j)$ for all $j \neq n$ with positive measure except for $t(n) \neq t'(n)$.

Let n' denote the next smaller firm size and n'' the next bigger firm size with respect to n with positive measure under t. By Lemma 1 and

Proposition 2 we know that matching is negative assortative. This implies that μ_{n_W} and $\mu_{n_W}^{strict}$ strictly increase in $t(n')$ and $t(n'')$. μ_{n_M} and $\mu_{n_M}^{strict}$ strictly decrease in $t(n')$ and $t(n'')$.

Suppose there exist side payments $t \neq t'$, both associated with a corresponding matching equilibrium such that conditions (16) hold.

Case $\{n : \mu_n > 0|t\} = \{n : \mu_n > 0|t'\}$: Let n, n', and n'' have positive measure for both t and t'. It follows that if $t(n) < t'(n)$, then $t'(n') > t(n')$ and $t'(n'') > t(n'')$ is necessary for μ_n, such that conditions (16) hold, to exist. An analogous argument applies to $t(n) > t'(n)$.

This implies that for $t' \neq t$ conditions (16) do not hold if $\{n : \mu_n > 0|t\} = \{n : \mu_n > 0|t'\}$. To see this, let μ_n, $\mu_{n'}$ and $\mu_{n''}$ be positive and suppose wlog. $t(n) < t'(n)$. Then μ_{n_M} and its strict version increase, $\mu_{n'_M}$ and $\mu_{n''_M}$ and their strict versions decrease. μ_{n_W} and its strict version decrease, and $\mu_{n'_W}$ and $\mu_{n''_W}$ and their strict versions increase. Increasing $t(n')$ or $t(n'')$ to induce measure consistency again the same effect appears on the next smaller and bigger firms with respect to n' and n'' firms. By induction all side payments in t' must increase. Hence, for the greatest firm size in equilibrium, the measure of agents willing to own must exceed the measure of agents willing to work, violating (16). If $|\{n : \mu_n > 0|t\}| < 3$ the argument can be applied accordingly.

Case $\{n : \mu_n > 0|t\} \cap \{n : \mu_n > 0|t'\} = \emptyset$: There exists n' with measure zero under t but positive measure under t' and n with positive measure under t but zero measure under t' such that n' is the next bigger or larger firm size with respect to n. This contradicts stability. To see this, note that stability of the equilibrium associated to t implies there is no $t'(n')$ under t such that n' is preferred to n by both workers and owners. For $t'(n')$, however, by stability of the equilibrium associated to t', n' is weakly preferred by a positive measure of both owners and workers to n for all side payments $t(n)$.

That is, a positive measure of both owners and workers must be indifferent between n under $t(n)$ and n' under $t'(n')$. Positive measures of both owners and workers for n firms under t must be measure consistent. However, indifference of both workers and owners for more than two consecutive firm sizes is impossible, since renegotiation payoffs induce linear independence in cutoff endowments by construction. Hence, measure consistency cannot be induced by an allocation with measure zero of n firms and positive measure of n' firms, a contradiction. This means $\{n : \mu_n > 0|t\} \subseteq \{n : \mu_n > 0|t'\}$ or $\{n : \mu_n > 0|t\} \supseteq \{n : \mu_n > 0|t'\}$.

Case $\{n : \mu_n > 0|t\} \supseteq \{n : \mu_n > 0|t'\}$: Suppose wlog. $t(n) < t'(n)$ for some n with positive measure under both t and t'. As shown previously this implies necessarily that $t(n') < t'(n')$ and $t(n'') < t'(n'')$. Suppose the next smaller or bigger firm size under t, say wlog. n', has zero measure under t'. To induce measure zero of n' firms t' has to be sufficiently greater than t for the labor supply in n' firms to collapse. But then agents preferring to own n' firms have to be matched in n firms and the next smaller n''' firms violating measure consistency for n firms. Restoring measure consistency for n''' and n firms requires side payments to rise in the next bigger and smaller firms. This implies by induction that for the biggest firm size under t' condition (16) cannot hold. This argument extends by induction to cases where more than one firm size has positive measure under t but zero measure under t'. Reversing the argument by exchanging t with t' gives the same result for $\{n : \mu_n > 0|t\} \subseteq \{n : \mu_n > 0|t'\}$.

Finally, the cases $t(n) = t'(n)$ for all $n \in \{n : \mu_n > 0|t\} \cap \{n : \mu_n > 0|t'\}$ either imply coincidence of side payments or can quickly be led to contradict stability.

Step 2: (P, θ) is unique with respect to choices (θ, n) in t almost everywhere. Only for $|\mu^{strict}_{n_M} - \mu_{n_M}| > 0$ and $|\mu^{strict}_{n_W} - \mu_n \leq \mu_{n_W}| > 0$ for $n = n', n$, with n the next bigger firm size than n', $\mu_n, \mu_{n'} > 0$, is the statement not trivial. But then measure consistency uniquely determines μ_n and $\mu_{n'}$, since to have $|\mu^{strict}_{n_M} - \mu_{n_M}| > 0$ and $|\mu^{strict}_{n_W} - \mu_n \leq \mu_{n_W}| > 0$ for more than two consecutive firm sizes is generically impossible, as four nonlinear equalities for cutoff endowments have to hold with three degrees of freedom (side payments in the three firms). $\square$

Proof of Proposition 4

Recall that $\pi_W(n') > \pi_W(n) \Leftrightarrow n > n'$ by Proposition 1. Determine first cutoff endowment levels for agents to prefer working in an efficient K firm as opposed to working (i) in an $n > K$ firm, in (ii) an $n' < K$ firm, or (iii) investing c to be part of an $n = 2$ firm.

$$\begin{aligned}
\omega(i) &\geq \frac{\pi_W(n) - \pi_W(K) + (1+r)\gamma(t(W,n) - t(W,K))}{(1+r)(\gamma-1)} - t(W,n), \\
\omega(i) &\leq \frac{\pi_W(K) - \pi_W(n') + (1+r)\gamma(t(W,K) - t(W,n'))}{(1+r)(\gamma-1)} - t(W,K), \\
\omega(i) &\leq \frac{\pi_W(K) - \pi_M(2) + (1+r)\gamma(c + t(W,K))}{(1+r)(\gamma-1)} - t(W,K). \qquad (17)
\end{aligned}$$

Abbreviate $t(n) := t(M,n)$ and conduct the same exercise for owners:

$$\begin{aligned}
\omega(i) &\leq \frac{\pi_M(K) - \pi_M(n) + (1+r)\gamma[t(K) - t(n) + (n-K)c]}{(1+r)(\gamma-1)} - t(K) + (K+1)c \\
\omega(i) &\geq \frac{\pi_M(n') - \pi_M(K) + (1+r)\gamma[t(n') - t(K) + (K-n')c]}{(1+r)(\gamma-1)} - t(n') + (n'+1)c \\
\omega(i) &\geq \frac{\pi_M(2) - \pi_M(K) + (1+r)\gamma((K-1)c - t(K))}{(1+r)(\gamma-1)} + c. \qquad (18)
\end{aligned}$$

We say an n firm *crowds out* an n' firm if, given equilibrium side payments $t(.,n)$, there do not exist side payments $t(.,n')$ such that there is both positive measure of agents preferring to be owner in n' firms and of agents preferring to be worker in n' firms to the same role in n firms.

Lemma 3 *A necessary condition for n firms to crowd out n' firms is*

$$t(W,n) - c < \frac{n'-1}{n'-n}\frac{\pi_W(n) - \pi_W(n')}{(1+r)\gamma} - \frac{1}{n'-n}\frac{\pi_M(n') - \pi_M(n)}{1+r}.$$

and for $n' > n > 1$ and

$$t(W,n) - c > \frac{n'-1}{n-n'}\frac{\pi_W(n') - \pi_W(n)}{1+r} - \frac{1}{n-n'}\frac{\pi_M(n) - \pi_M(n')}{(1+r)\gamma}.$$

and for $1 < n' < n$.

Proof of Lemma: Assume first that $n < n'$. Then, by Lemma 2 for n firms to crowd out n' firms, given $t(n)$ there must not exist $t(n')$ such that

$$\omega_M(n,n') \leq n'c - t(n') \text{ and } \omega_W(n,n') \geq t(W,n'),$$

where $\omega_M(n,n')$ and $\omega_W(n,n')$ is the appropriate cutoff endowment level as derived in (18) and (17). That is, $\nexists\, t(n') = -(n'-1)t(W,n')$ (because of budget balance and Lemma 1) such that

$$\frac{\pi_W(n') - \pi_W(n)}{(1+r)\gamma} + \frac{t(n)}{n-1} - \frac{t(n')}{n'-1} \geq 0 \text{ and}$$
$$\frac{\pi_M(n) - \pi_M(n')}{1+r} + t(n) - t(n') + (n'-n)c \leq 0.$$

Solving for $\frac{t(n')}{n'}$ yields the conditions in the lemma for $n' > n$ and $n' < n$. □

By the lemma, efficient firms crowd out $n > K$ ($n' < K$) firms only if

$$\frac{t(K)}{K-1} - c \;<\; \frac{n-1}{n-K}\frac{\pi_W(K) - \pi_W(n)}{(1+r)\gamma} - \frac{\pi_M(n) - \pi_M(K)}{(n-K)(1+r)} \text{ and}$$
$$\frac{t(K)}{K-1} - c \;>\; \frac{n'-1}{K-n'}\frac{\pi_W(n') - \pi_W(K)}{1+r} - \frac{\pi_M(K) - \pi_M(n')}{(K-n')(1+r)\gamma},$$

Obtaining bounds on $t(K)$ by comparing the efficient firm size with symmetric $n = 1$ firms yields

$$\frac{\pi_M(2) - \pi_M(K)}{(K-1)(1+r)} \leq \frac{t(K)}{K-1} - c \leq \frac{\pi_W(K) - \pi_M(2)}{(1+r)\gamma}.$$

Now it is possible to derive the desired necessary conditions. For all $n > K$ and $n' < K$ it must hold that

$$\frac{n-1}{n-K}\frac{\pi_W(K) - \pi_W(n)}{\gamma} - \frac{\pi_M(n) - \pi_M(K)}{n-K} + \frac{\pi_M(K) - \pi_M(2)}{K-1} \;>\; 0 \text{ and}$$
$$\gamma\frac{n'-1}{K-n'}[\pi_W(n') - \pi_W(K)] + \frac{\pi_M(n') - \pi_M(K)}{K-n'} + \pi_M(2) - \pi_W(K) \;<\; 0. \quad (19)$$

Rewriting (19) yields

$$\frac{f(K)}{K-1} - \frac{f(n)}{n-1} - \frac{(n-K)\pi_M(2)}{(n-1)(K-1)} - \frac{\gamma-1}{\gamma}[\pi_W(K) - \pi_W(n)] > 0 \text{ and} \quad (20)$$
$$(\gamma-1)(n'-1)[\pi_W(n') - \pi_W(K)] + f(n') - f(K) + (K-n')\pi_M(2) < 0. \quad (21)$$

The necessary conditions in the propositions follow. Condition (21) holds for γ sufficiently close to 1 but the LHS of (21) strictly increases in γ. Moreover, the LHS of (21) decreases in the average output of K firms, $\frac{f(K)}{K}$, all else equal. It remains inconclusive whether the LHS of condition (20) increases or decreases in γ for $\gamma \gg 1$ and increases in $\frac{f(K)}{K}$. This means for sufficiently high γ the necessary conditions fail given the production function. □

Proof of Proposition 5

To have positive measure of both $n = K+1$ and $n' = K-1$ in equilibrium,

(i) size K firms must not crowd out n nor n' firms, that is conditions (19) must fail for n and n'.

(ii) n and n' firms must not crowd out each other and therefore conditions (19) where K is substituted by n' and n, respectively, must fail to hold. Rewriting these conditions as in inequalities (20) and (21) yields

$$\frac{f(n')}{n'-1} - \frac{f(n)}{n-1} - \frac{\gamma-1}{\gamma}\left[\pi_W(n') - \pi_W(n)\right] - \frac{(n-n')\pi_M(2)}{(n-1)(n'-1)} \leq 0 \text{ and}$$
$$(\gamma-1)(n'-1)[\pi_W(n') - \pi_W(n)] + n'\frac{f(n')}{n'} - n\frac{f(n)}{n} + (n-n')\pi_M(2) \geq 0.$$

Both conditions always hold for sufficiently high γ and sufficiently high average output of n firms compared to n' firms. Additionally we need that $\underline{\omega}$ is sufficiently small and $\overline{\omega}$ sufficiently large, as $\underline{\omega} < \omega_W(n,n') < \omega_M(n,n') < \overline{\omega}$.

(iii) there must not exist any other firm size h crowding out n or n' firms as in Lemma 3. That is, the appropriate versions of (20) and (21) have to hold for both n and n'. Setting $n = K+1$ and $n' = K-1$ and using (i) and (ii) it suffices to check that $K+1$ firms are not crowded out by any $h > K+1$ firm and $K-1$ firms are not crowded out by any $h' < K-1$ firm leading to the conditions in the proposition. $\square$

Conditions for the Example

Neither $K+1$ nor $K-1$ firms crowd out each other:

$$\begin{aligned}
\frac{\gamma-1}{\gamma}\Delta\pi_W(K-1,K+1) &\geq (K-2)^{-\frac{1}{K}} - K^{-\frac{1}{K}} - \frac{1}{K(K-2)}, \\
(\gamma-1)\Delta\pi_W(K-1,K+1) &\geq \frac{K^{1-\frac{1}{K}}}{K-2} - (K-2)^{-\frac{1}{K}} - \frac{1}{K-2}.
\end{aligned}$$

Smaller firms do not crowd out $K-1$ firms: For $j = 2, ..., K-2$

$$(\gamma-1)\Delta\pi_W(K-j,K-1) \leq \frac{(K-2)^{1-\frac{1}{K}}}{K-j-1} - (K-j-1)^{-\frac{1}{K}} - \frac{(j-1)}{2(K-j-1)}.$$

Larger firms do not crowd out $K+1$ firms: For $j = 2, ..., \overline{n} - K$

$$\frac{\gamma-1}{\gamma}\Delta\pi_W(K+1,K+j) \leq K^{-\frac{1}{K}} - (K+j-1)^{-\frac{1}{K}} - \frac{j-1}{2K(K+j-1)}.$$

These conditions are independent of w_0. Setting $K = 4$ and $\gamma = 2.5$ (or e.g. $K = 5$ and $\gamma = 3$) a calculation exercise verifies that all inequalities hold. Hence, equilibrium measures of both size 3 and 5 firms will be positive, if some agents are wealthy enough to prefer to own size 5 firms. $\mu(i \in I : \omega(i) \geq 5c) > 0$ is sufficient, since $\omega^*(M,4,M,5) < 5c$. Higher values of γ, e.g. $\gamma > 3.5$, allow us to conclude that size 2 and 6 firms emerge.

Proof of Proposition 6

Let (P^F, θ^F, t^F) be an equilibrium under endowment distribution F, and G be a counterclockwise rotation of F as in Definition 3. Denote by (P^G, θ^G, t^G) the equilibrium under G. Since $\hat{\omega} > c$ agent i with $\omega_i^F = \hat{\omega}$ is member of a size 2 or manager in a size $\hat{n}$ firm. Choose $\omega' = \omega_U^F$ and $\omega'' = \omega^{F^*}(W,\hat{n},W,n^+)$ where $n^+ > \hat{n}$ denotes the next higher firm size with positive measure. As

$$\mu(\omega^F(i) \leq \omega) \leq \mu(\omega^G(i) \leq \omega)\ \forall\, \omega > \hat{\omega},$$

we have that $\sum_{n=\hat{n}+1}^{\overline{n}} \mu_{n_M}|_{F,t^F} \geq \sum_{n=\hat{n}+1}^{\overline{n}} \mu_{n_M}|_{G,t^F}$.[18] In particular, for $\overline{n}^F > \hat{n}$ defined as the biggest firm size with $\mu_{\overline{n}^F}|_F > 0$, it must hold that $\mu_{\overline{n}^F{}_M}|_G < \mu_{\overline{n}^F{}_M}|_F$. Likewise, as additionally

$$\mu(\omega^F(i) \leq \omega) \geq \mu(\omega^G(i) \leq \omega)\ \forall\, \omega < \hat{\omega},$$

and matching is negative assortative, for all $m > \hat{n}$ we have $\sum_m^{\overline{n}^F} \mu_{W_n}|_{F,t^F} \geq \sum_m^{\overline{n}^F} \mu_{W_n}|_{G,t^F}$ if $F(\omega^{F^*}(W,\hat{n},W,n^+)) - F(\omega_U^F) \geq G(\omega^{F^*}(W,\hat{n},W,n^+)) - G(\omega_U^F)$.

Then $\sum_{n>\hat{n}} \mu_n|_{F,t^F} \geq \sum_{n>\hat{n}} \mu_n|_{G,t^F}$ for unchanged side payments t^F. Note that equality only holds if F and G coincide for all agents in $n > 1$ firms. This means both supply and demand for $n > \hat{n}$ workers weakly decrease. Equating supply and demand using side payments t^G then gives measures $\mu_n|_{G,t^G}$ for which it must hold that $\sum_{n=\hat{n}+1}^{\overline{n}} \mu_n|_{G,t^G} \leq \sum_{n=\hat{n}+1}^{\overline{n}} \mu_n|_{F,t^F}$ and for the biggest firm size $\overline{n}$ in particular that $\mu_{\overline{n}}|_{G,t^G} < \mu_{\overline{n}}|_{F,t^F}$. Moreover, $F(\omega_U|_{F,t^F}) \geq G(\omega_U|_{G,t^G})$ since either $\overline{n}$ workers are scarce in which case side payments (and ω_U) decrease, or $\overline{n}$ owners are scarce in which case $F(\omega_U|_{F,t^F}) < G(\omega_U|_{G,t^G})$ implies that $\mu_{\overline{n}}|_{G,t^G} > \mu_{\overline{n}}|_{F,t^F}$, contradicting the statement above. $\square$

[18] We follow the convention of writing $\mu_{M_n}|_{F,t^F}$ to indicate the measure of individuals weakly preferring to be owners of n firms given side payments t^F and the endowment distribution F.

Proof of Proposition 7

Monotonicity of incomes in endowments follows from a revealed preferences argument. Suppose agent j chooses (θ, n) in equilibrium and agent i with $\omega(i) \neq \omega(j)$ chooses $(\theta', n') \neq (\theta, n)$ in equilibrium. Let without loss of generality upfront investments be greater for $t(\theta', n')$. Since both optimize

$$\begin{aligned} v(\omega(i), \theta, n, t(\theta, n), r) &\leq v(\omega(i), \theta', n', t(\theta', n'), r) \text{ and} \\ v(\omega(j), \theta, n, t(\theta, n), r) &\geq v(\omega(j), \theta', n', t(\theta', n'), r). \end{aligned} \tag{22}$$

Since investment is greater for (θ', n') and capital cost depends on wealth, by Lemma 2 inequalities (22) imply $\omega(j) < \omega(i)$. Then $v(\omega(i), \theta, n, t(\theta, n), r) > v(\omega(j), \theta, n, t(\theta, n), r)$ because $\gamma > 1$, and equilibrium incomes $v(.)$ are indeed increasing in endowments.

Turn now to income and endowment gaps for agents matched into n firms. Denote by j the poorest agent matched into an n firm. j's income is given by

$$v(\omega(j), W, n, t(W, n), r) = \pi_W(n) + (1 + r)\gamma(\omega(j) + t(W, n)),$$

assuming $-t(W, n) > \omega(j)$. Let i denote the richest owner of an n firm. Then

$$v(\omega(i), M, n, t(M, n), r) = \pi_M(n) + (1 + r)\gamma(\omega(i) + t(M, n) - nc),$$

assuming $\omega(i) < nc - t(M, n)$. The ratio of incomes is then

$$\frac{v(\omega(i), M, n, .)}{v(\omega(j), W, n, .)} = \frac{(1+r)\gamma\omega(i) + \pi_M(n) - (1+r)\gamma(nc + (n-1)t(W, n))}{(1+r)\gamma\omega(j) + \pi_W(n) + (1+r)\gamma t(W, n)}. \tag{23}$$

Note at this point that for an unmatched agent h choosing the outside option $(W, 1)$ we have that $v(\omega(i), \theta, n, t(M, n), r)/v(\omega(h), W, 1, 0, r) \geq \omega(i)/\omega(h)$ for any matched agent i, since i's continuation payoff is at least as high as i's outside option. If both agents borrow, a necessary and sufficient condition for

$$v(\omega(i), M, n, t(M, n), r)/v(\omega(j), W, n, t(W, n), r) < \omega(i)/\omega(j)$$

is

$$\pi_M(n) - (1+r)\gamma[nc + (n-1)t(W,n)] < \pi_W(n) + (1+r)\gamma t(W,n). \quad (24)$$

This implies that for sufficiently low side payments such that $\pi_M(n) < (1+r)\gamma[nc + (n-1)t(W,n)]$ the above inequality holds trivially. j prefers to be worker, so that

$$\pi_W(n)+(1+r)\gamma(\omega(i)+t(W,n)) > \pi_M(n)+(1+r)\gamma[\omega(i)-nc-(n-1)t(W,n)],$$

which immediately implies (24). This argument extends to the other cases. Note that if j lends and i borrows j's income can be written as $v(\omega(j),.) = (1+r)\gamma\omega(j) - (i-r)\omega(j) + (1+r)t(W,n)$. □

Proof of Proposition 8

A bimodal firm size distribution implies there exists a trough at some size n

$$\mu_{n^+} \geq \mu_n > 0 \text{ and } \mu_{n^-} \geq \mu_n > 0 \text{ with } n > 2, \quad (25)$$

where n^+ (n^-) is the next higher (smaller) firm size with positive measure implying the support of ω is large enough to permit heterogeneous firms. That is,

$$\min\left\{\frac{\mu_{n_W^+}}{n^+}; \mu_{n_M^+}\right\} > \mu_n \text{ and } \min\left\{\frac{\mu_{n_W^-}}{n^-}; \mu_{n_M^-}\right\} > \mu_n.$$

Suppose $\omega^*(M,n,n^-) \leq nc - t(M,n) < \omega^*(M,n^+,n)$,[19] then a necessary condition for $\mu_{n^+} > \mu_n$ is

$$\begin{aligned} &\mu[\omega^*(M,n^{++},n^+) \geq \omega(i) \geq \omega^*(M,n^+,n)] \\ &> \mu[\omega^*(M,n^+,n) \geq \omega(i) \geq \omega^*(M,n,n^-)], \end{aligned} \quad (26)$$

where n^{++} denotes the next higher firm size with positive measure compared to n^+. If n^{++} does not exist, substitute $\omega^*(M,n^{++},n^+) = \overline{\omega}$. The differences

[19]We abbreviate notation when the occupational role does not change.

in cutoff endowment are given as

$$\omega^*(M,n^{++},n^+) - \omega^*(M,n^+,n) = \frac{2\pi_M(n^+) - \pi_M(n^{++}) - \pi_M(n)}{(1+r)(\gamma-1)}$$
$$- \frac{((\gamma+1)n^+ - \gamma n^{++} - n)c}{\gamma - 1} + \frac{(\gamma+1)t(M,n^+) - \gamma t(M,n^{++}) - t(M,n)}{\gamma-1}.$$

and

$$\omega^*(M,n^+,n) - \omega^*(M,n,n^-) = \frac{2\pi_M(n) - \pi_M(n^+) - \pi_M(n^-)}{(1+r)(\gamma-1)}$$
$$- \frac{((\gamma+1)n - \gamma n^+ - n^-)c}{\gamma - 1} + \frac{(\gamma+1)t(M,n) - \gamma t(M,n^+) - t(M,n^-)}{\gamma-1}.$$

Using the fact that n^-, n, and n^+ have positive measure, i.e. are not crowded out by each other or size 2 firms, Lemma 3 can be applied to yield upper and lower bounds $\underline{\kappa}$ and $\overline{\kappa}$ not depending on side payments such that

$$\omega^*(M,n^{++},n^+) - \omega^*(M,n^+,n) < \overline{\kappa} \text{ and } \omega^*(M,n^+,n) - \omega^*(M,n,n^-) > \underline{\kappa}.$$

Decreasing endowment density and (26) then imply there is $\kappa_1 = \underline{\kappa}/\overline{\kappa}$ with

$$\frac{\mu[\omega^*(M,n^{++},n^+) \geq \omega(i) \geq \omega^*(M,n^+,n)]}{\mu[\omega^*(M,n^+,n) \geq \omega(i) \geq \omega^*(M,n,n^-)]} > \kappa_1 \frac{\omega^*(M,n^{++},n^+) - \omega^*(M,n^+,n)}{\omega^*(M,n^+,n) - \omega^*(M,n,n^-)}.$$

The same argument applies to workers in n^+ firms when assuming $\omega^*(W,n,n^+) \leq t(M,n)/(n-1) < \omega^*(W,n^-,n)$ and comparing the roles (W,n^+) and (W,n) yielding (iii).

Suppose that $nc - t(M,n) = \omega^*(M,n,n^+)$ and $t(M,n)/(n-1) = \omega^*(W,n^-,n)$. Since n^-, n, and n^+ firms have positive measure $t(M,n^+) \leq (n^+ - 1)\omega^*(W,n,n^+)$ and $n^-c - t(M,n^-) \leq \omega^*(M,n^-,n)$. Combining all these facts contradicts decreasing differences of $f(n)$. Hence, either (ii) or (iii) or both have to hold. $\square$

Parametrization of the Numerical Examples

Let $c = 0.15$ and $\gamma = 3$. The technology has the form $f(n) = \hat{f}(n) + nw_0$ with

$$\hat{f}(n) = \left((n-1)h^2\right)^{0.68 - 0.06\sqrt{|n-K|}},$$

where $K = 4$ and $h = 0.9$. The wealth distribution is

$$\phi(\omega) = \frac{1}{G}\left[\frac{1}{2}\omega(g\omega^{g-l}\exp(-g\omega) + (1-\frac{1}{2}\omega))(\beta\lambda^{\beta}\omega^{\beta-1}\exp(-(\lambda\omega)^{\beta}))\right],$$

where G satisfies $\int_{\underline{\omega}}^{\overline{\omega}}\phi(\omega)d\omega = 1$. Further details generating the figures in the paper are given in tables 1 and 2.

Parameters	Figure 2	Figure 3	Results	Figure 2	Figure 3
r	0.0000	0.0000	mean wealth	0.1477	0.1477
$\underline{\omega}$	0.0000	0.0000	$n = 2$ firms	0.0764	0.0862
$\overline{\omega}$	1.0000	1.0000	$n = 3$ firms	0.0664	0.0709
β	1.2500	2.9000	$K = 4$ firms	0.0460	0.0621
λ	11.9920	10.6000	$n = 5$ firms	0.0290	0.0490
g	1.3000	2.5000	$n = 6$ firms	0.0532	0.0181
l	1.5500	2.9000	mean output	0.4235	0.4285

Table 1: Simulation Details for Figures 2 and 3

Parameters	before	after	Results	before	after
r	0.1900	0.1900	mean wealth	0.1479	0.1498
$\underline{\omega}$	0.0000	0.0125	$n = 2$ firms	0.0826	0.0695
$\overline{\omega}$	1.0000	1.0125	$n = 3$ firms	0.0673	0.0613
β	1.9000	1.3000	$K = 4$ firms	0.0546	0.0430
λ	10.4500	13.0000	$n = 5$ firms	0.0284	0.0269
g	2.9000	2.2000	$n = 6$ firms	0.0432	0.0604
l	3.6000	2.5000	mean output	0.4216	0.4174

Table 2: Simulation Details for Figure 4

The algorithm starts by assigning a value to $t(3)$ and setting all other side payments to make workers indifferent between firm sizes. A step begins with the largest firm size that has positive labor supply or demand. Equalize supply and demand for that firm size (possibly at zero) by varying side payments in smaller firm sizes such that workers remain indifferent between smaller firm sizes. Take into account that managers may have to be made indifferent between different firm sizes. Repeat this for firm sizes $n > 3$ in descending order. This gives a residual excess labor demand at $n = 3$. Choose a new value for $t(3)$ using the residual excess labor demand by a version of Newton's method with dampening and start a new step. The algorithm stops when the residual excess labor demand equals zero, again taking into account that managers may have to be made indifferent. Convergence of the algorithm can be shown using the proof of uniqueness. The simulation source code can be obtained from the author or on his homepage.

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